70: Discrete Math and Probability Theory

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Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

The truth: My hopes and dreams.

You learn to think more clearly and more powerfully.

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You learn to think more clearly and more powerfully.

..And to deal precisely clearly with uncertainty itself.

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And to tell the truth.

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Truth??

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Truth?? Is there truth?

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Truth?? Is there truth? Evidence to decisions.

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Truth?? Is there truth? Evidence to decisions. What are values?

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Mathematical Reasoning is as close to truth as there is.

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It has a certain context.

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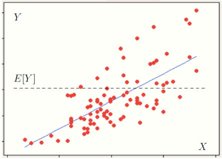
And the context vast consequences.

Probability Unit

- How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, …)?
  - Constructive Models: Model the overall system (including the sources of uncertainty).
    - For modeling uncertainty, we'll develop probabilistic models and techniques for analyzing them.
  - Deductive Models: Extract the "trend" from the previous outcomes (e.g., linear regression).



Learning.

Veritassium on Khan

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Confusion is the sweat of learning.

Learning.

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Confusion is the sweat of discovery.

Metacogition.

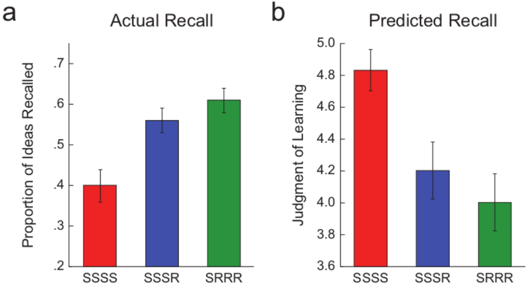

Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students' metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students' actual long-term retention.

Learning styles.

How to search google.

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"Learning styles"

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  "Learning styles debunked."

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CS70: Notes, lectures, discussions, vitamins, homeworks.

An effective student is...

Smart, rich,

An effective student is...

Smart, rich,

and beautiful.

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and beautiful.

All memes. The last one is not a meme.
First one, learning is inherent.

An effective student is...

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First one, learning is inherent. You are all capable.

Second, background, background, etc.

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Confident, motivated,

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What I think.

Confident, motivated,

has integrity.

Known knowns..

There are the known knowns, known unknowns, and unknown unknowns.

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The last one is what always gets you.

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In order to get there.

Dogs don’t have rights cuz..

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They don't know infinity.

First grade

$1, 2, 3, 4, \ldots, 120$

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Peano's axioms. There is always a successor.

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$+3$ means move to successor and another and another, or 3 times.

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Metric. (distance.) There is mapping $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$
Obeys triangle inequality: $f(i,j) + f(j,k) \geq f(i,k)$

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3×5?

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3×5?

$\times$ means add 3 times.
$5 + 5 + 5$
10 is moving over 5 from 5
The next number one can use the one's place.

How to interact with staff..

My advice to TA's.

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What should you do?

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What should you do?
Where does your understanding get iffy?

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E.g., Avoid long explanations with nodding students. You must checkin meaningfully.

What should you do?
  Where does your understanding get iffy?
    Explain what you understand, then say what you don't.

Advice from (former) TA's

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Head TA Richard:
"carefully review the homework solutions after they are released and understand them to the point of being able to replicate them without needing to reference them."

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See this paper, for example and a good discussion.

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Please do not take it out on your TA's.

Admin

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Explains policies, has office hours, homework, midterm dates, etc.

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One midterm, final.

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Questions

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Questions $\Longrightarrow$ Ed:

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Questions $\Longrightarrow$ Ed:
Logistics, etc.
Content Support: other students!
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It's weekly.

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It's weekly.
Read it!!!!

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Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
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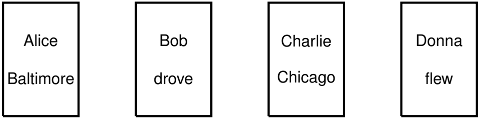
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.



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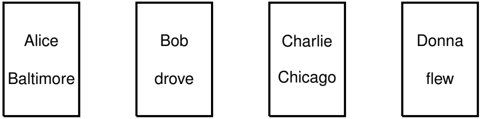

- Which cards must you flip to test the theory?

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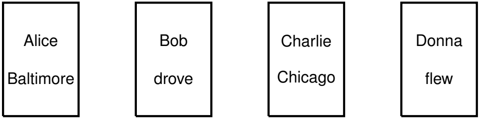

- Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D).

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side, and mode of travel.
- Consider the theory:
  "If a person travels to Chicago, they flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Alice	Bob	Charlie	Donna
Baltimore	drove	Chicago	flew

- Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D). Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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Today: Note 1.   Note 0 is background.

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Today: Note 1. Note 0 is background. Do read it.

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The language of proofs!

CS70: Lecture 1. Outline.

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
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Any even > 2 is sum of 2 primes
$4 + 5$
$x + x$
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Propositional Forms.

Put propositions together to make another...

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Propositions:

P_1 - Person 1 rides the bus.

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We can program!!!!

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We need a way to keep track!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
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Check: $\wedge$ and $\vee$ are commutative.

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One use for truth tables: Logical Equivalence of propositional forms!

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$.

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True if $\geq$ one of P or Q is True .

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F	T	T
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Check: $\wedge$ and $\vee$ are commutative.

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P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	

Truth Tables for Propositional Forms.

"$P \land Q$" is True if both P and Q are True .

P	Q	$P \land Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \lor Q$" is True if $\geq$ one of P or Q is True .

P	Q	$P \lor Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

P	Q	$\neg(P \lor Q)$	$\neg P \land \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

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"$P \vee Q$" is True if $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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"$P \vee Q$" is True if $\geq$ one of P or Q is True .

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T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$$

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True if $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q)$

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if both P and Q are True .

P	Q	$P \wedge Q$
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T	T	T
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Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ $\qquad$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes!

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:
P is True .
LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.
RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.
P is False .
LHS: $F \land (Q \lor R) \equiv F$.
RHS: $(F \land Q) \lor (F \land R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.

P is False .

LHS: $F \land (Q \lor R) \equiv F$.

RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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True Statements: P, $P \implies Q$.

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Examples:

Implication.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

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Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
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$P \implies Q$ interpreted as

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Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"

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If P, then Q.

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Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

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Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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The statement "$P \implies Q$"

only is False if P is True and Q is False .

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Implication and English.

$P \implies Q$

Poll.

- If P, then Q.

Implication and English.

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Poll.

- If P, then Q.
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Implication and English.

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  Remember if P is true then Q must be true.

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- P is sufficient for Q.

Implication and English.

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  Example: Showing $n > 4$ is sufficient for showing $n > 3$.

Implication and English.

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  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.

  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Implication and English.

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  This means that proving P allows you
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  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.
  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

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T	T	T
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P	Q	$\neg P \vee Q$
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T	F	
F	T	
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$\neg P \vee Q \quad \equiv P \implies Q.$

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$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

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Logically equivalent! Notation: $\equiv$. Recall: $(X \implies Y) \equiv (\neg X \vee Y)$

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- **Converse** of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

Contrapositive, Converse

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  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!) converse!
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  Logically equivalent! Notation: $\equiv$. Recall: $(X \implies Y) \equiv (\neg X \vee Y)$
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- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
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  - If you stand in the rain, you get wet.
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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Next: Statements about boolean valued functions!!

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- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
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Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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$$(\forall x \in N)\,(2x > x) \qquad \textbf{False} \qquad \textbf{Consider } x = 0$$

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- "Square of any natural number greater than 5 is greater than 25."

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Later we may omit universe if clear from context.

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English: there is an x in S where $P(x)$ does not hold.

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For False , find x, where $\neg P(x)$.

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Counterexample.

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Theorem: $(\forall n \in \mathbb{N})\ n \geq 3 \implies \neg(\exists a, b, c \in \mathbb{N})\ (a^n + b^n = c^n)$

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Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

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DeMorgan Restatement:
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Summary.

Propositions are statements that are true or false.

Propositional forms use $\wedge, \vee, \neg$.

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Next Time: proofs!