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Last time:

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  Shared (and sort of kept) secrets.

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Today: Errors

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  Tolerate Loss: erasure codes.

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Modular Arithmetic Fact: Exactly 1 polynomial of degree $\leq d$ with arithmetic modulo prime p contains $d + 1$ pts.

Proof sketches.

Property 2 A polynomial: $P(x) = a_d x^d + \cdots a_0$ has $d+1$ coefficients.

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Multiply by zero. My love is won.
Combine.

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Property 1 A non-zero degree d polynomial has at most d roots.

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Love me some contradiction!
Two polynomials: $P(x), Q(x)$, $P(x) - Q(x)$ has too many roots.

Finite Fields

Proof works for reals, rationals, and complex numbers.

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In the rationals, the precision blows up, where in modular arithmetic, it does not.

Secret Sharing

Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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n people, k is enough.

(A) The modulus needs to be at least $n+1$.
(B) The modulus needs to be at least k.
(C) Use degree k polynomial, hand out n points.
(D) Use degree n polynomial, hand out k points.
(E) Use degree $k-1$ polynomial, hand out n points.
(F) The modulus needs to be at least 2^s, where s is value of secret.
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(A), (B), (E), (F)

Erasure Codes.

Satellite

GPS device

Erasure Codes.

Satellite

3 packet message.

GPS device

Erasure Codes.

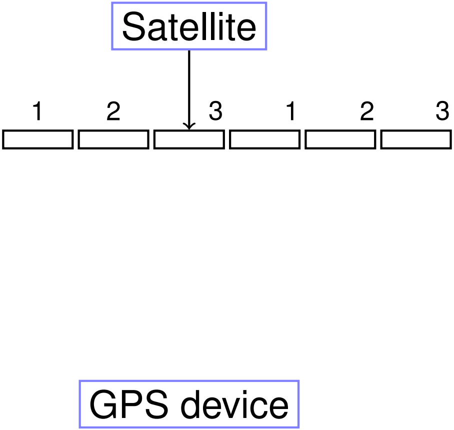
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

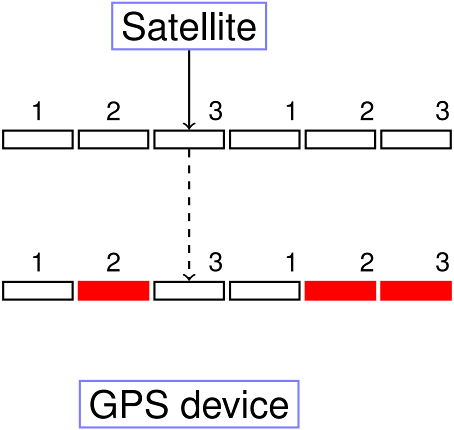
Erasure Codes.



3 packet message. So send 6!

Lose 3 out 6 packets.

Erasure Codes.



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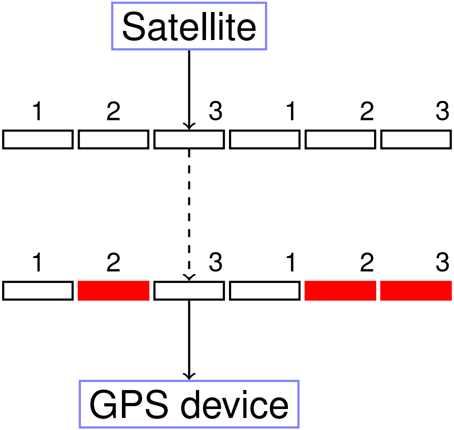

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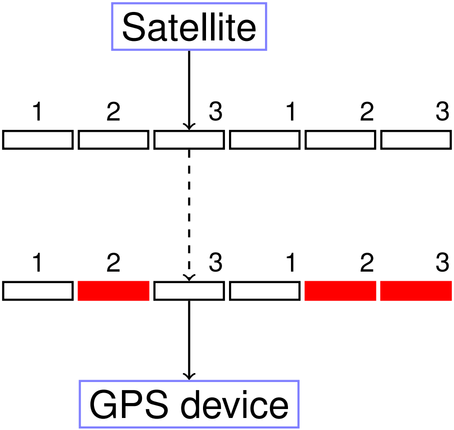

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Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Use polynomials.

The Scheme

Problem: Want to send a message with n packets.

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A degree $n-1$ polynomial determined by any n points!

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1. Choose prime $p \approx 2^b$ for packet size b.
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GPS device

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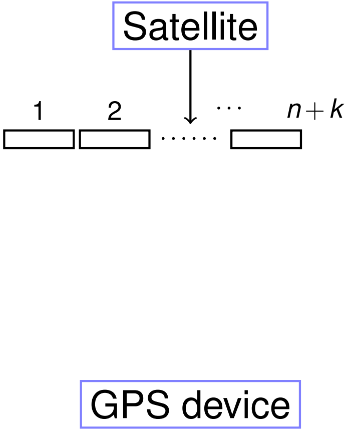

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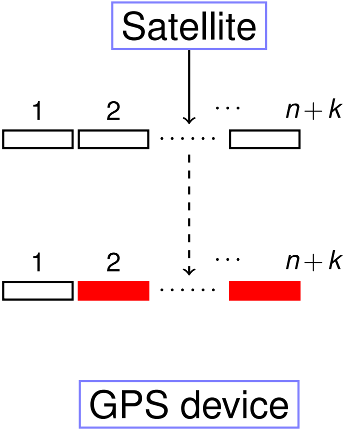

n packet message. So send $n+k$!

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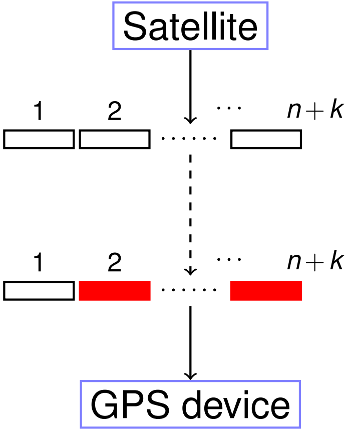

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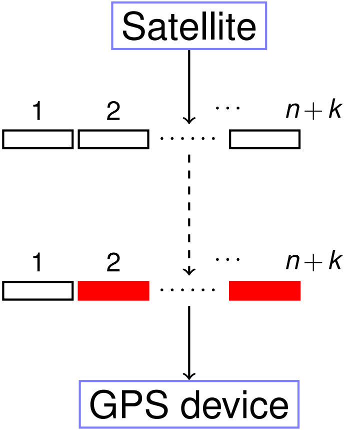

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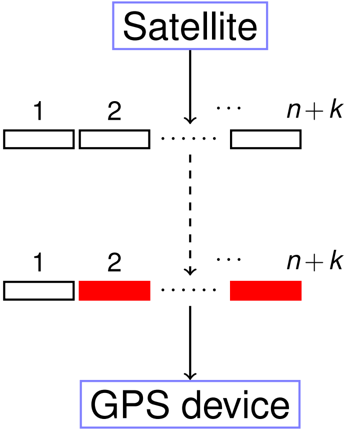

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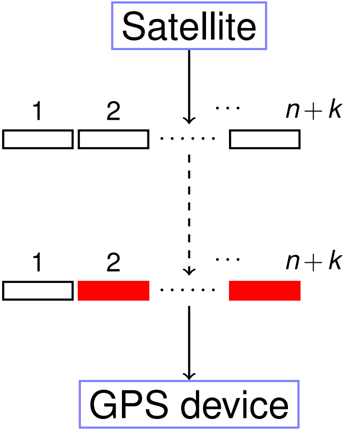

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Optimal.

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Modulo 7 to accommodate at least 6 packets.

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Notice that packets contain “x-values”.

Bad reception!

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Bad reception!

Send: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

Recieve: (1,1) (2,4), (6,0)
Reconstruct?

Format: $(i, R(i))$.

Lagrange or linear equations.

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You want to encode a secret consisting of 1,4,4.

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Modulus should be larger than $n+k$ and also larger than 2^b.

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- ..give Secret Sharing.

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Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

Error Correction

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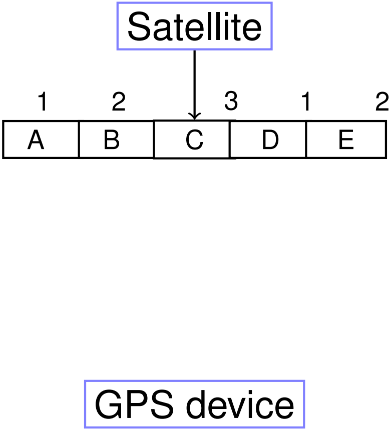
3 packet message.

Corrupts 1 packets.

GPS device

Error Correction



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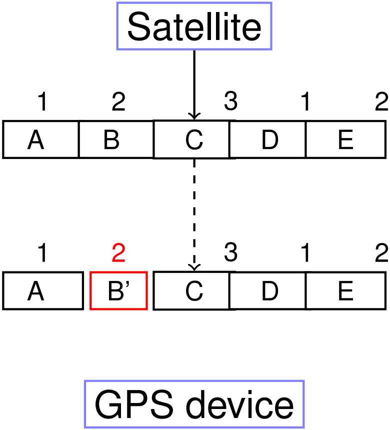

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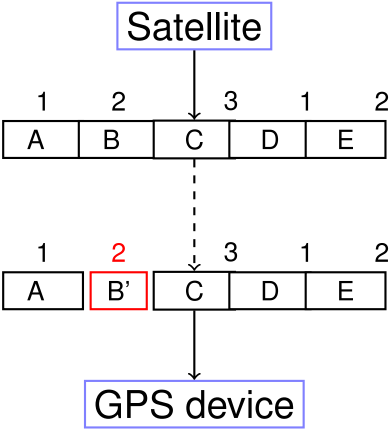

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Problem: Communicate n packets $m_1, \ldots, m_n$
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Properties: proof.

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$Q(x) = R(i)$, on set of size $n+k$. $\mathscr{Q} = \{i : Q(i) = R(i)\}$ $P(x) = R(i)$, on set of size $n+k$. $\mathscr{P} = \{i : P(i) = R(i)\}$

$\bar{\mathscr{Q}} \cup \bar{\mathscr{P}} \leq 2k \implies \mathscr{Q} \cap \mathscr{P} \geq n$
$\implies P(i) = R(i) = Q(i)$ on $\geq n$ values of i's
$\implies Q(i) = P(i)$ at n points. $\implies Q(x) = P(x)$. $\square$

Example.

Message: $3, 0, 6$.

Example.

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Reed Solomon Code:

Example.

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Reed Solomon Code:

$P(x) = x^2 + x + 1 \pmod 7$

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Example.

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Example.

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(Aside: Message in plain text!)

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Send: $P(1) = 3, P(2) = 0, P(3) = 6, P(4) = 0, P(5) = 3$.

(Aside: Message in plain text!)

Receive $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$.

$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

Slow solution.

Brute Force:
For each subset of $n+k$ points
Fit degree $n-1$ polynomial, $Q(x)$, to n of them.

Slow solution.

Brute Force:
For each subset of $n+k$ points
  Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
  Check if consistent with $n+k$ of the total points.

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Brute Force:
For each subset of $n+k$ points
  Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
  Check if consistent with $n+k$ of the total points.
  If yes, output $Q(x)$.

Slow solution.

Brute Force:
For each subset of $n+k$ points
Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
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If yes, output $Q(x)$.

- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!

Slow solution.

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For each subset of $n+k$ points
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Check if consistent with $n+k$ of the total points.
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- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!
- For any subset of $n+k$ pts,
  1. unique degree $n-1$ polynomial $Q(x)$ that fits $\geq n$ of them

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For each subset of $n+k$ points
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- For any subset of $n+k$ pts,
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  2. and where $Q(x)$ is consistent with $n+k$ points

Slow solution.

Brute Force:
For each subset of $n+k$ points
Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
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  $\Longrightarrow P(x) = Q(x)$.

Slow solution.

Brute Force:
For each subset of $n+k$ points
Fit degree $n-1$ polynomial, $Q(x)$, to n of them.
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- For subset of $n+k$ pts where $R(i) = P(i)$, method will reconstruct $P(x)$!
- For any subset of $n+k$ pts,
  1. unique degree $n-1$ polynomial $Q(x)$ that fits $\geq n$ of them
  2. and where $Q(x)$ is consistent with $n+k$ points

  $\implies P(x) = Q(x)$.

Reconstructs $P(x)$ and only $P(x)$!!

Example.

Send: $P(1) = 3, P(2) = 0, P(3) = 6,$

Example.

Send: $P(1) = 3, P(2) = 0, P(3) = 6, P(4) = 0, P(5) = 3.$

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Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Example.

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

$$
\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
4p_2 + 2p_1 + p_0 &\equiv 1 \pmod 7 \\
2p_2 + 3p_1 + p_0 &\equiv 6 \pmod 7 \\
2p_2 + 4p_1 + p_0 &\equiv 0 \pmod 7 \\
4p_2 + 5p_1 + p_0 &\equiv 3 \pmod 7
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Assume point 1 is wrong

Example.

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Assume point 1 is wrong and solve..

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Assume point 1 is wrong and solve..no consistent solution!

Example.

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\end{aligned}$$

Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong

Example.

Send: $P(1) = 3, P(2) = 0, P(3) = 6, P(4) = 0, P(5) = 3.$

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

$$\begin{aligned} p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\ 4p_2 + 2p_1 + p_0 &\equiv 1 \pmod 7 \\ 2p_2 + 3p_1 + p_0 &\equiv 6 \pmod 7 \\ 2p_2 + 4p_1 + p_0 &\equiv 0 \pmod 7 \\ 4p_2 + 5p_1 + p_0 &\equiv 3 \pmod 7 \end{aligned}$$

Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong and solve...

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Assume point 1 is wrong and solve..no consistent solution!
Assume point 2 is wrong and solve...consistent solution!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Error!!

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Error!! .... Where???

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Error!! .... Where???
Could be anywhere!!!

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

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Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!.

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Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!.

How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
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And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
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Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong

Ditty...

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Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
Zero times anything is zero!!!!!

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All equations satisfied!!!!!

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But which equations should we multiply by 0?

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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$E(i) = 0$ if and only if $e_j = i$ for some j

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All equations satisfied!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

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In general:

Example.

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Plugin points...

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

But don't know error locator polynomial!

Do know form: $(x - e)$.

In general: $x^k + b_{k-1} x^{k-1} + \cdots b_0$.

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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In general: $x^k + b_{k-1}x^{k-1} + \cdots b_0$.

also: $(x - e_1)(x - e_2)\ldots(x - e_k)$.

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Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

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4 unknowns (p_0, p_1, p_2 and e),

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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
&\vdots \\
(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod{p} \\
&\vdots \\
(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
\end{aligned}
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$$\begin{aligned} E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\ &\vdots \\ E(i)(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i)E(i) \pmod{p} \\ &\vdots \\ E(m)(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m)E(m) \pmod{p} \end{aligned}$$

...so satisfied, I'm on my way.

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and linear in a_i and coefficients of $E(x)$!

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Solving for $Q(x)$ and $E(x)$...

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           ------------------
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```

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          -----------------
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```
                 1 x^2 + 1 x
          -----------------
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         -----------------
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         ----------
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                 ---------------
                           x + 5
```

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```
                1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
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         ----------
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                ---------------
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```
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        -----------------
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                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x + 1
          -----------------
    x - 2 ) x^3   + 6 x^2 + 6 x + 5
            x^3   - 2 x^2
            ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$

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                1 x^2 + 1 x + 1
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    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                  1 x^2 + 1 x + 1
           -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
                  ---------------
                            x + 5
                            x - 2
                            -----
                                0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$?

Example: finishing up.

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                 1 x^2 + 1 x + 1
        -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

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What is $\frac{x-2}{x-2}$? 1

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x + 1
         ------------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.   Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Proof:
We claim

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Equation 2 implies 1:

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
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$E(x)$ and $E'(x)$ have at most k zeros each.

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Can cross divide at n points.

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Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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for $i \in \{1, \ldots n+2k\}$.

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If $E(i) = 0$, then $Q(i) = 0$.

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If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

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Cross multiplying gives equality in fact for these points.

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Points to polynomials, have to deal with zeros!

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. □

Points to polynomials, have to deal with zeros!

Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Poll

Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1),...P(8)$.

You recieve packets $R(1),...R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
(B) The degree of $P(x)E(x) = 3+2 = 5$.
(C) The degree of $E(x)$ is 2.
(D) The number of coefficients of $P(x)$ is 4.
(E) The number of coefficients of $P(x)Q(x)$ is 6.

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(E) is false.

(A) $E(x) = (x-1)(x-4)$
(B) The number of coefficents in $E(x)$ is 2.
(C) The number of unknown coefficents in $E(x)$ is 2.
(D) $E(x) = (x-1)(x-2)$
(E) $R(4) \neq P(4)$
(F) The degree of $R(x)$ is 5.

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(E) $R(4) \neq P(4)$
(F) The degree of $R(x)$ is 5.

(A), (C), (E). (F) doesn't type check!

Summary. Error Correction.

Communicate n packets, with k erasures.

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Communicate n packets, with k erasures.

How many packets?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n + k$

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.

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Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree?

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover?

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

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Communicate n packets, with k errors.

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Communicate n packets, with k errors.

How many packets? $n+2k$
Why?

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How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

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How many packets? $n+2k$
Why?
k changes to make diff. messages overlap
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Reed-Solomon codes.

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Reed-Solomon codes. Welsh-Berlekamp Decoding.

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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!