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Say you sent a message of length 4, encoded as $P(x)$ where one sends packets $P(1),...P(8)$.

You recieve packets $R(1),...R(8)$.

Packets 1 and 4 are corrupted.

(A) $R(1) \neq P(1)$
(B) The degree of $P(x)E(x) = 3+2 = 5$.
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two polynomials "correct on" $n+k$ overlap on n points
Two polynomials of degree $n-1$ mean same polynonmial.
Recover?
Reconstruct error polynomial, $E(X)$, and $P(x)$!
Nonlinear equations.
Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$. Linear Equations.
Polynomial division!

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
Why?
k extra "good" packets after k errors.
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
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Reed-Solomon codes.

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Reed-Solomon codes. Welsh-Berlekamp Decoding.

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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Cool.

Really Cool!

Probability

What’s to come?

Probability

What's to come? Probability.

Probability

What's to come? Probability.

A bag contains:

Probability

What's to come? Probability.

A bag contains:

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

For now:

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

For now: Counting!

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

For now: Counting!

Later: Probability.

The future in this course.

What's to come?

The future in this course.

What's to come? Probability.

The future in this course.

What's to come? Probability.

A bag contains:

The future in this course.

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Count blue. Count total.

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

(A) Red Probability is $3/8$
(B) Blue probability is $3/9$
(C) Yellow Probability is $2/8$
(D) Blue probability is $3/8$

The future in this course.

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(A) Red Probability is $3/8$
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Today:

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide. **Chances?**

(A) Red Probability is $3/8$
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Today: Counting!

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a n sided convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?
How many ways can I divide up 5 dollars among 3 people?

Using a tree..

How many 3-bit strings?

Using a tree..

How many 3-bit strings?

How many different sequences of three bits from $\{0,1\}$?

Using a tree..

How many 3-bit strings?
  How many different sequences of three bits from $\{0, 1\}$?
  How would you make one sequence?

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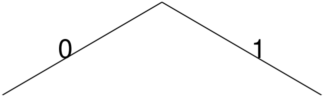

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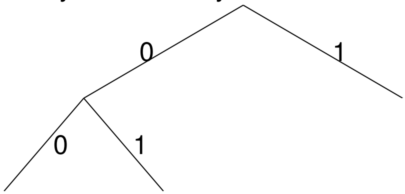

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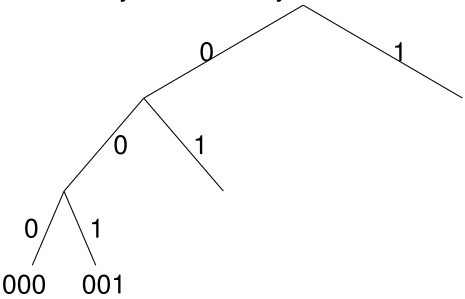

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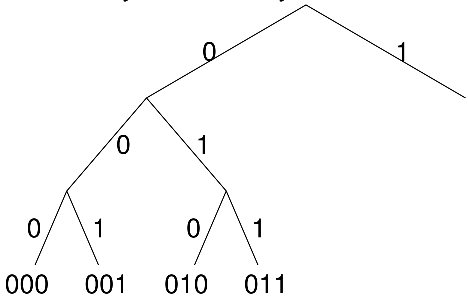

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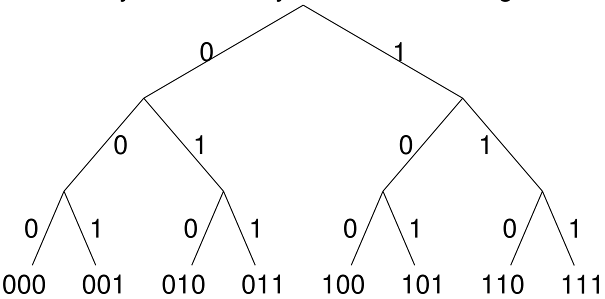

8 leaves which is $2 \times 2 \times 2$.

Using a tree..

How many 3-bit strings?
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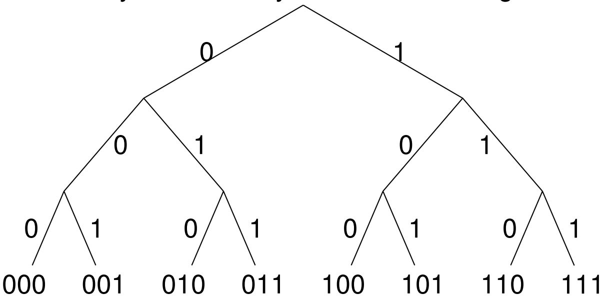

8 leaves which is $2 \times 2 \times 2$.   One leaf for each string.

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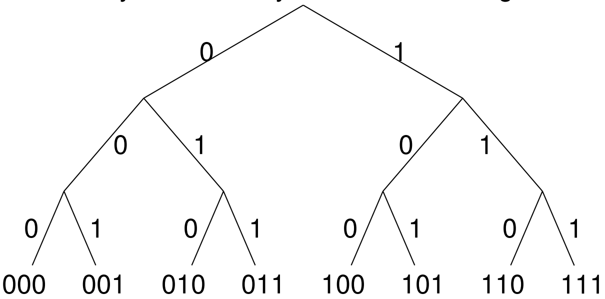

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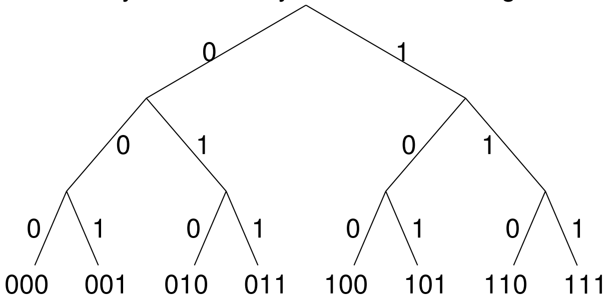

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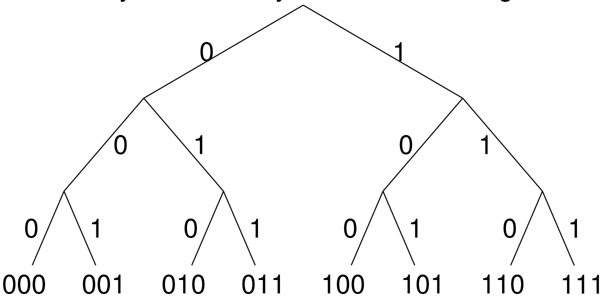

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

8 3-bit strings! Also recurrence: $B(n) = 2 \times B(n-1)$.

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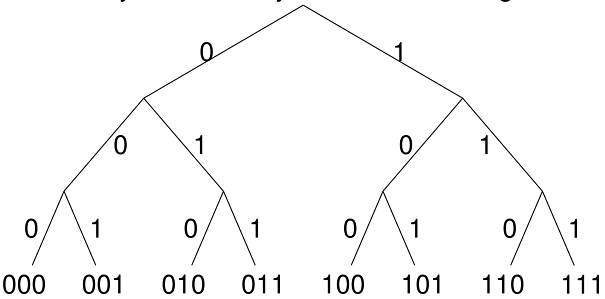

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

8 3-bit strings! Also recurrence: $B(n) = 2 \times B(n-1)$. $B(0) = 1$.

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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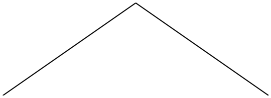

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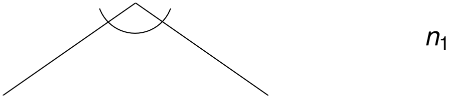

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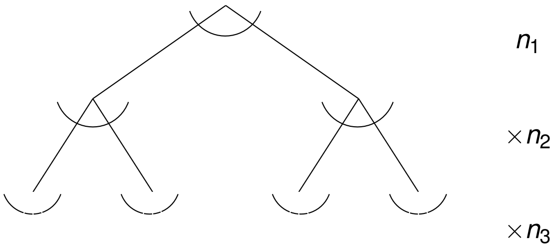

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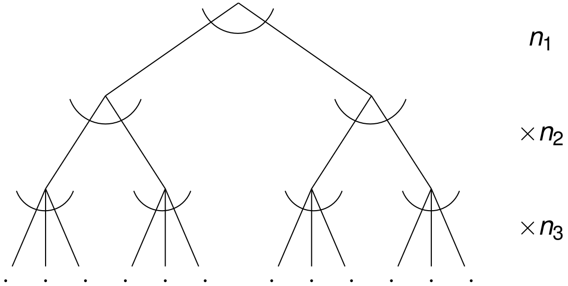

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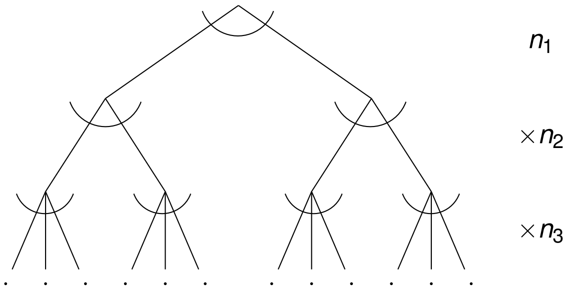

In picture, $2 \times 2 \times 3 = 12$!

First Rule of Counting: Product Rule

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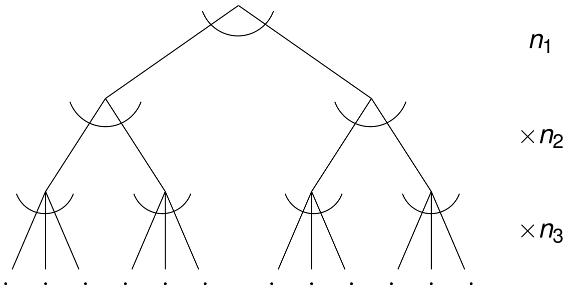
the number of objects is $n_1 \times n_2 \cdots \times n_k$.



In picture, $2 \times 2 \times 3 = 12$!
Also recurrence: $S(i) = n_i \times S(i+i)$, $B(k+1) = 1$

Poll

Mark whats corect.

Poll

Mark whats corect.

(A) $|\text{10 digit numbers}| = 10^{10}$
(B) $|k \text{ coin tosses}| = 2^k$
(C) $|\text{10 digit numbers}| = 9 * 10^9$
(D) $|n \text{ digit base } m \text{ numbers}| = m^n$
(E) $|n \text{ digit base } m \text{ numbers}| = (m-1)m^{n-1}$

Poll

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(E) $|n \text{ digit base } m \text{ numbers}| = (m-1)m^{n-1}$

(A) or (C)? (D) or (E)? (B) are correct.

Using the first rule..

How many outcomes possible for k coin tosses?

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

Using the first rule..

How many outcomes possible for k coin tosses?

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How many 10 digit numbers?

Using the first rule..

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How many 10 digit numbers?

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
10

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
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How many 10 digit numbers?

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How many 10 digit numbers?

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$10 \times 10 \cdots \times 10$

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How many n digit base m numbers?

Using the first rule..

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

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$10 \times 10 \cdots \times 10 = 10^k$

How many n digit base m numbers?

m ways for first,

Using the first rule..

How many outcomes possible for k coin tosses?

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How many 10 digit numbers?

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How many n digit base m numbers?

m ways for first, m ways for second, ...

Using the first rule..

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$10 \times 10 \cdots \times 10 = 10^k$

How many n digit base m numbers?

m ways for first, m ways for second, ...
m^n

Using the first rule..

How many outcomes possible for *k* coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^k$

How many *n* digit base *m* numbers?

m ways for first, *m* ways for second, ...
m^n

(Is 09, a two digit number?)

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

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$10 \times 10 \cdots \times 10 = 10^k$

How many n digit base m numbers?

m ways for first, m ways for second, ...
m^n

(Is 09, a two digit number?)

If no. Then $(m-1)m^{n-1}$.

Functions, polynomials.

How many functions f mapping S to T?

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$|T|$ ways to choose for $f(s_1)$,

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How many polynomials of degree d modulo p?

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How many polynomials of degree d modulo p?

p ways to choose for first coefficient,

Functions, polynomials.

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How many functions f mapping S to T?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...

....$|T|^{|S|}$

How many polynomials of degree d modulo p?

p ways to choose for first coefficient, p ways for second, ...
...p^{d+1}

p values for first point, p values for second, ...
...p^{d+1}

Questions?

Permutations.

[1] By definition: $0! = 1$.

Permutations.

How many 10 digit numbers **without repeating a digit**?

---

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How many different samples of size *k* from *n* numbers **without replacement.**

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

---

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Are $A, K, Q, 10, J$ of spades
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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

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Number of orderings for a poker hand: "5!"

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(The "!" means factorial, not Exclamation.)

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$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

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Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

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Counting sets..when order doesn't matter.

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Number of orderings for a poker hand: "5!"

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$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

---

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Ordered to unordered.

Second Rule of Counting: If order doesn’t matter count ordered objects and then divide by number of orderings.

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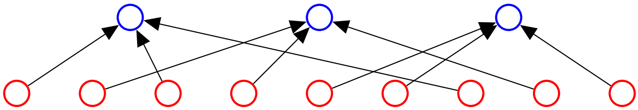

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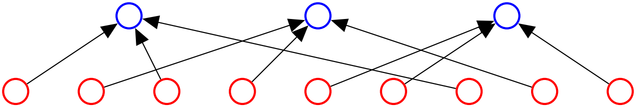

How many red nodes (ordered objects)?

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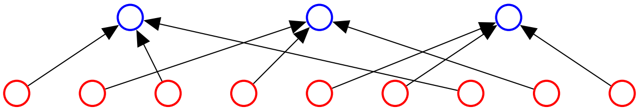

How many red nodes (ordered objects)? 9.

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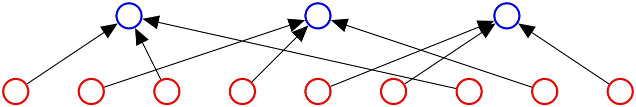

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

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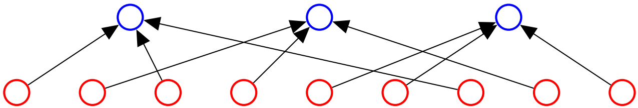

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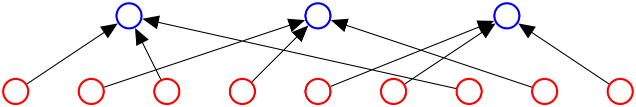

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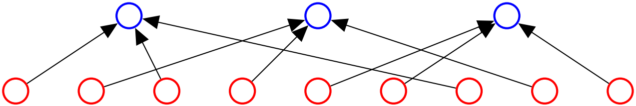

How many red nodes (ordered objects)? 9.

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How many blue nodes (unordered objects)? $\frac{9}{3}$

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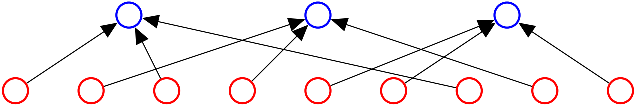

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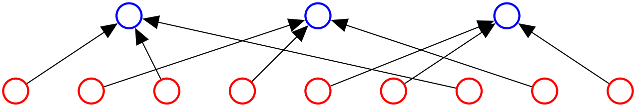

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How many poker deals?

Ordered to unordered.

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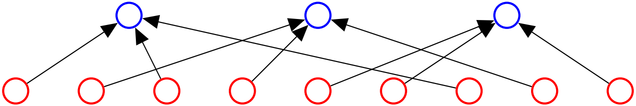

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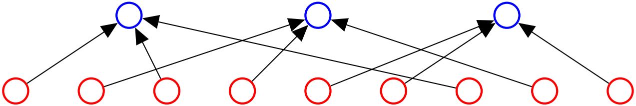

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How many poker deals per hand?

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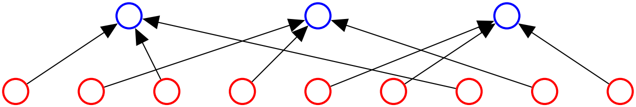

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How many poker deals per hand?
Map each deal to ordered deal:

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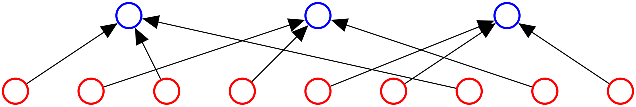

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How many poker deals per hand?
  Map each deal to ordered deal: $5!$

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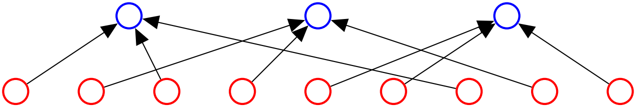

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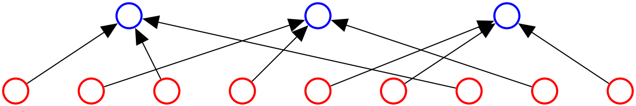

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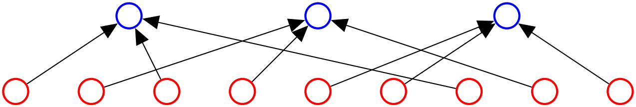

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Questions?

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Choose 2 out of n?

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$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)!}$$

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Notation: $\binom{n}{k}$ **and pronounced "n choose k."**

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Familiar?

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Choose 3 out of n?

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$$\frac{n!}{(n-k)! \times k!}$$

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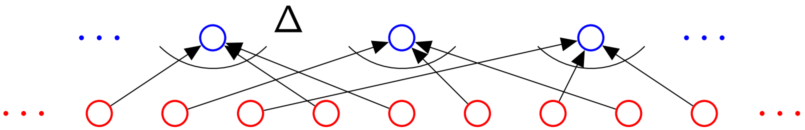
Familiar? Questions?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



Example: Visualize the proof..

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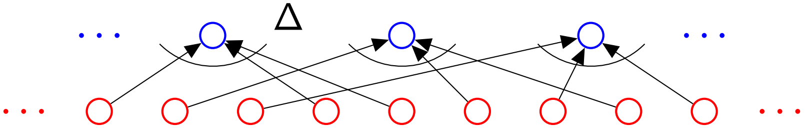
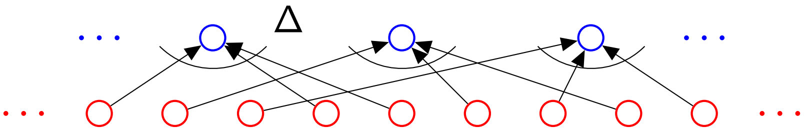


3 card Poker deals: 52

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: 52×51

Example: Visualize the proof..

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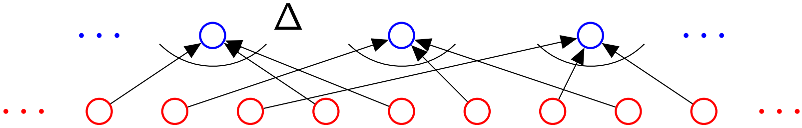
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50$

Example: Visualize the proof..

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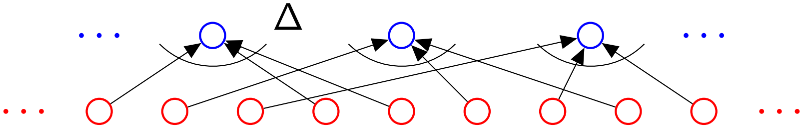
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$.

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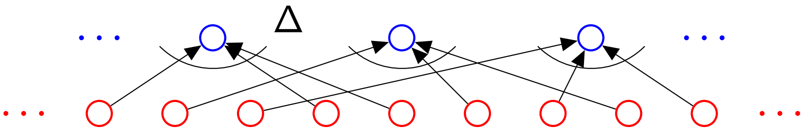

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.

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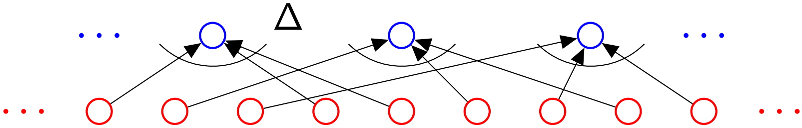

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?

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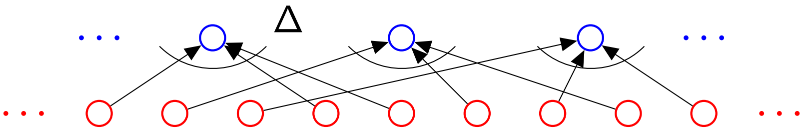

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.

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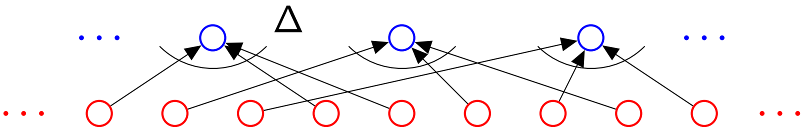

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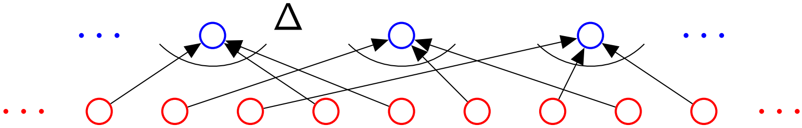

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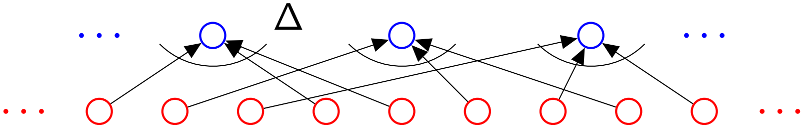

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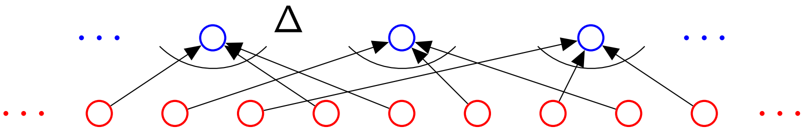

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$\Delta = 3 \times 2 \times 1$

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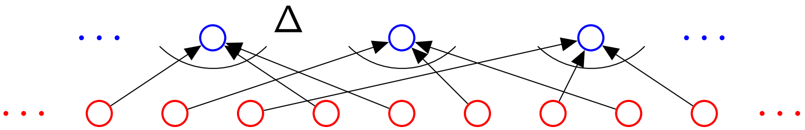

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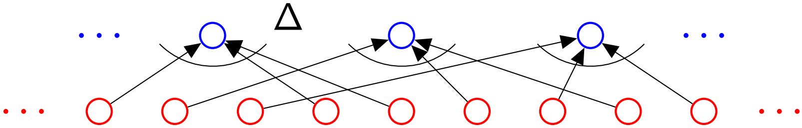

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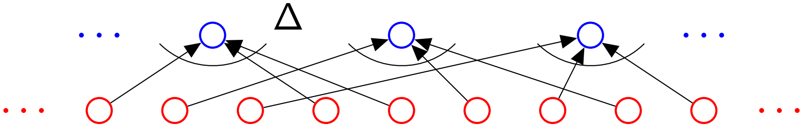

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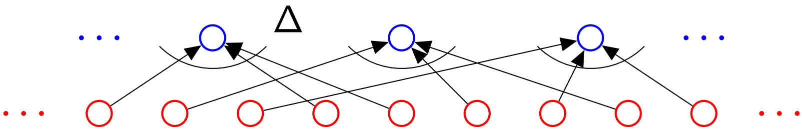

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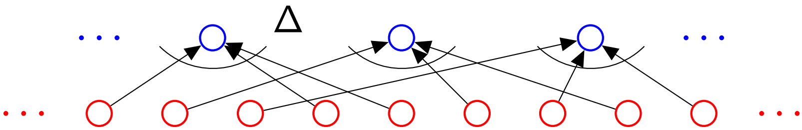

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Choose k out of n.

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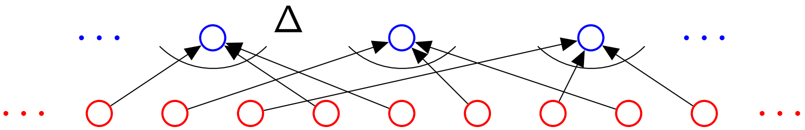

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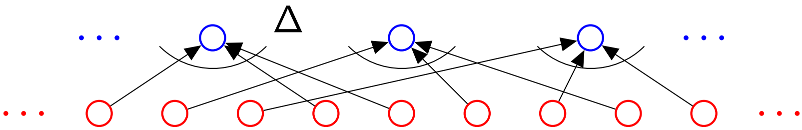

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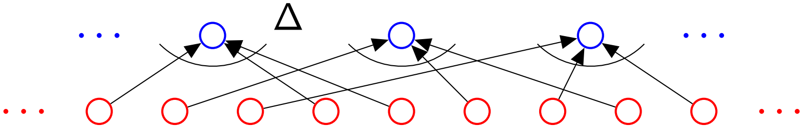

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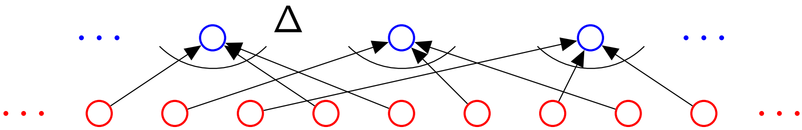

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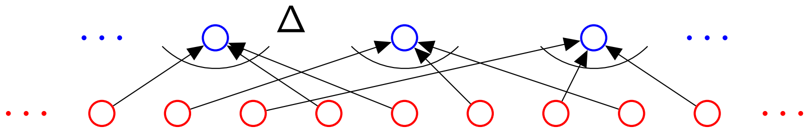

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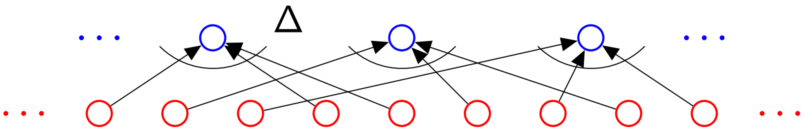

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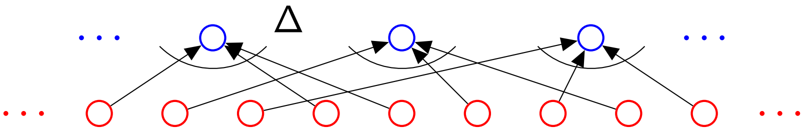

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Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
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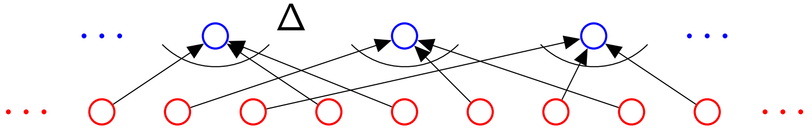

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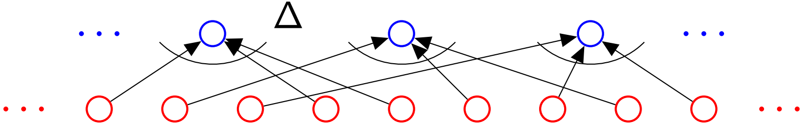

Orderings of ANAGRAM?

Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
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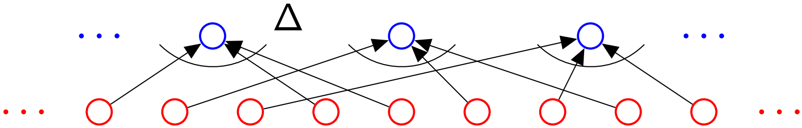

Orderings of ANAGRAM?
Ordered Set: 7!

Example: Anagram

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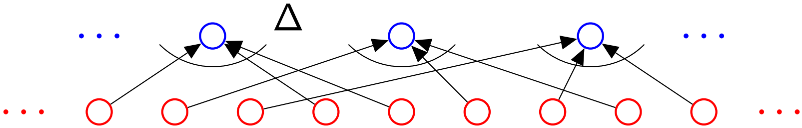

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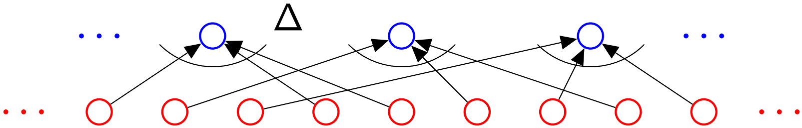

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A's are the same!

Example: Anagram

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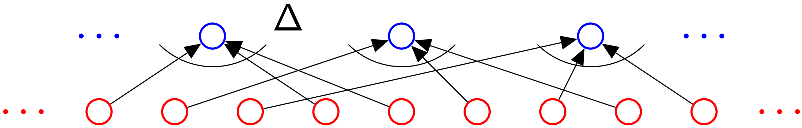

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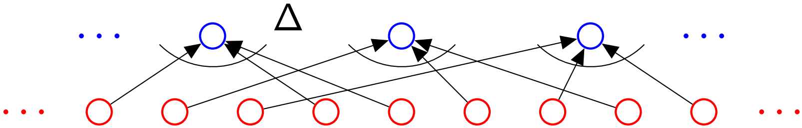

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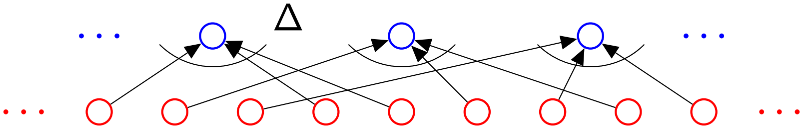

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ANAGRAM
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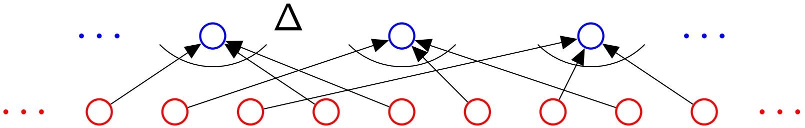

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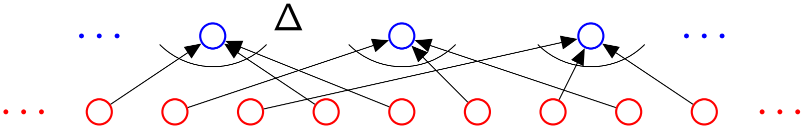

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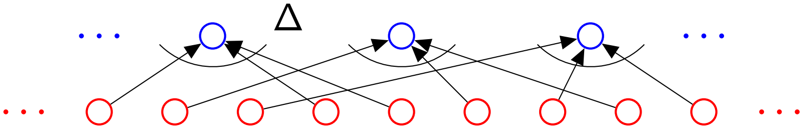

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ANAGRAM

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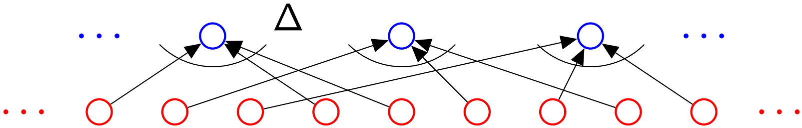

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Example: Anagram

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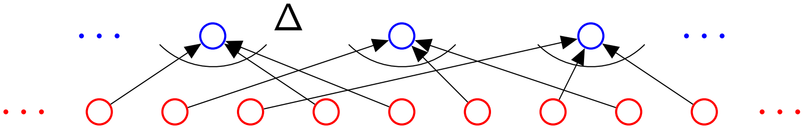

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Ordered Set: 7! First rule.

A's are the same!

What is Δ?

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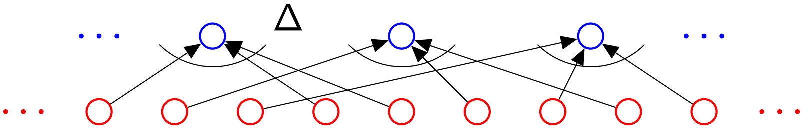

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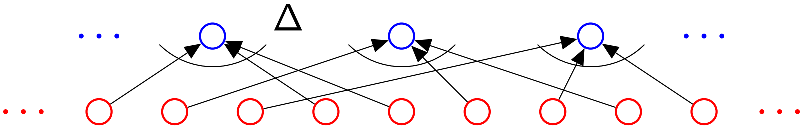

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Poll

Mark what's correct.

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(A) $|\text{Poker hands}| = \binom{52}{5}$
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(A)-(E) are correct.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Sampling...

Sample k items out of n

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Without replacement:

Sampling...

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Without replacement:

Order matters:

Sampling...

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Without replacement:

Order matters: $n \times$

Sampling...

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Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

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Order matters: $n \times n$

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(B, B, B, B, B):
(A, B, B, B, B):
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and so on.

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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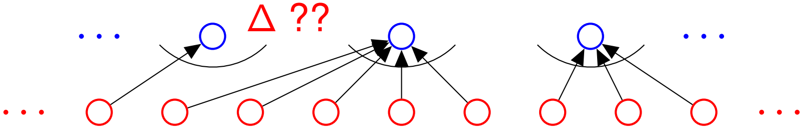

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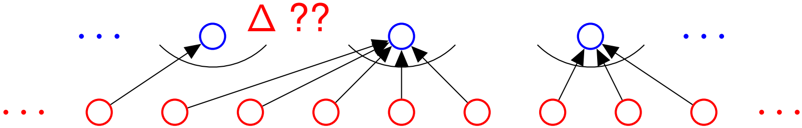

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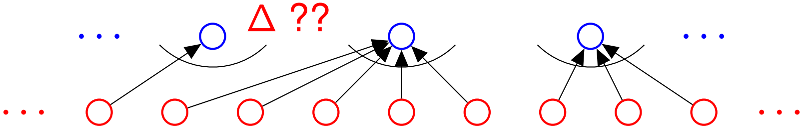

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(A, A, B, B, B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...

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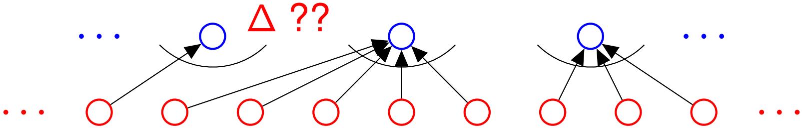

Second rule of counting is no good here!

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

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Each sequence of stars and bars "is" a split.

Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$| \star | \star \star \star \star$.

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Bars in first and third position.

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Bars in second and seventh position.

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$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

Stars and Bars.

Ways to add up n numbers to sum to k?

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$n+k-1$ positions from which to choose $n-1$ bar positions.

$$\binom{n+k-1}{n-1}$$

Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Poll

Mark whats correct.

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(A) ways to split k dollars among n: $\binom{k+n-1}{n-1}$

(B) ways to split n dollars among k: $\binom{n+k-1}{k-1}$

(C) ways to split 5 dollars among 3: $\binom{5+3-1}{3-1}$

(D) ways to split 5 dollars among 3: $\binom{7}{5}$

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All correct.

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