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Construct "diagonal" number: .7

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Construct "diagonal" number: .77

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Subset $[0,1]$ is not countable!!

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Any subset of a countable set is countable.

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If reals are countable then so is $[0, 1]$.

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1. Assume that a set S can be enumerated.

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6. Contradiction.

Another diagonalization.

The set of all subsets of N.

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The set of all subsets of *N*.

Example subsets of *N*: $\{0\}$,

Another diagonalization.

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There is a listing, *L*, that contains all subsets of *N*.

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Theorem: The set of all subsets of N is not countable.
(The set of all subsets of S, is the **powerset** of N.)

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Mark parts of proof.

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(A) Integers are larger than naturals cuz obviously.
(B) Integers are countable cuz, interleaving bijection.
(C) Reals are uncountable cuz obviously!
(D) Reals can't be in a list: diagonal number not on list.
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(B), (C)?, (D), (E)

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There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

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There is no infinite set whose cardinality is between the cardinality of an infinite set and its power set.

The powerset of a set is the set of all subsets.

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If math doesn't contain a contradiction.

This statement is a lie.

Is the statement above true?

The barber shaves every person who does not shave themselves.

Who shaves the barber?

Self reference.

Can a program refer to a program?

Can a program refer to itself?

Uh oh....

The Barber!

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(A) Barber not Mark. Barber shaves Mark.
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Its all true.

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Its all true. It's a problem.

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Recall: powerset of the naturals is not countable.

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Any set of axioms is either

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See Logicomix by Doxiaidis, Papadimitriou (was professor here), Papadatos, Di Donna.

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Write me a program checker!

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Run P on I and check!

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How long do you wait?

Something about infinity here, maybe?

Halt does not exist.

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What is he talking about?

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Professor does me some love Welch-Berlekamp though!

Halt and Turing.

Proof:

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Proof: Assume there is a program *HALT*$(\cdot, \cdot)$.

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Proof: Assume there is a program $HALT(\cdot, \cdot)$.

Turing(P)

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Proof: Assume there is a program $HALT(\cdot,\cdot)$.

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1. If HALT(P,P) =“halts”, then go into an infinite loop.

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Does Turing(Turing) halt?

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$\implies$ then HALTS(Turing, Turing) $=$ halts
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Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Another view of proof: diagonalization.

Any program is a fixed length string.

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P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Programs?

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(A) Instructions.
(B) Text.
(C) Binary String.
(D) They run on computers.

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All are correct.

Proof play by play.

Assumed HALT(P, I) existed.

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We are so smart!

Wow, that was easy!

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Wow, that was easy!

We should be famous!

No computers for Turing!

In Turing’s time.

No computers for Turing!

In Turing's time.

No computers.

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No computers.

Adding machines.

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e.g., Babbage (from table of logarithms) 1812.

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Concept of program as data wasn't really there.

Turing machine.

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A Turing machine.
– an (infinite) tape with characters

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– be in a state, and read a character

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Turing: AI, self modifying code, learning...

Turing and computing.

Just a mathematician?

Turing and computing.

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Turing and computing.

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“Wrote” a chess program.

Simulated the program by hand to play chess.

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  The polish machine...the *bomba.*

Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

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We can't get enough of building more Turing machines.

Undecidable problems.

Does a program, *P*, print “Hello World”?

Undecidable problems.

Does a program, P, print "Hello World"?
How?

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Does a program, P, print "Hello World"?
How? What is P?

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Can a set of notched tiles tile the infinite plane?

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Proof: simulate a computer. Halts if finite.

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Undecidability for Diophantine set of equations
$\implies$ no program can take any set of integer equations and

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The answer is yes or no. This "problem" is not undecidable.

Undecidability for Diophantine set of equations
$\implies$ no program can take any set of integer equations and always corectly output whether it has an integer solution.

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Turing: personal.

Tragic ending...

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- British Government apologized (2009) and pardoned (2013).

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Program is text, so we can pass it to itself,

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Program is text, so we can pass it to itself,
or refer to self.

Summary: decidability.

Computer Programs are an interesting thing.

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Computation is a lens for other action in the world.

Kolmogorov Complexity, Google, and CS70

Of strings, s.

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Minimum sized program that prints string s.

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What Kolmogorov complexity of a string of 1,000,000, one's?

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for $i = 1$ to n: print '1'.

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What is the minimum I need to know (remember) to know stuff.

Kolmogorov Complexity, Google, and CS70

What is the minimum I need to know (remember) to know stuff.

Radius of the earth?

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Radius of the earth? Distance to the sun?

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Solution to: $dy/dx = y$,

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Calculus:

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Depends on your skills!

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Calculus: what is minimum you need to know?
Depends on your skills! Conceptualization.
Reason and understand an argument and you can generate a lot.

Calculus

What is the first half of calculus about?

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The slope of a tangent line to a function at a point.

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Chain rule?

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Idea: use rise in function value!

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Perhaps.

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$\sin(x)$.

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What is x? An angle in radians.

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Let's call it θ and do derivative of $\sin\theta$.

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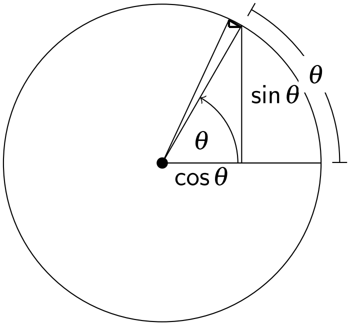

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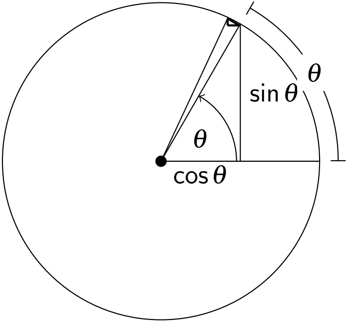

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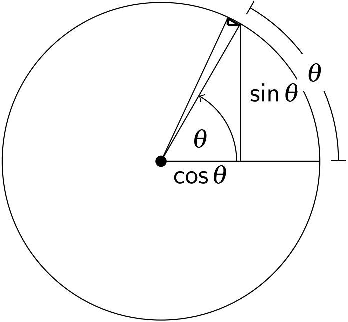

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Similar triangle!!!

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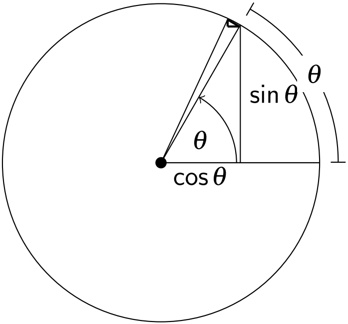

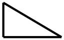

Rise.
Similar triangle!!!
Rise proportional to cosine!

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

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Useful?

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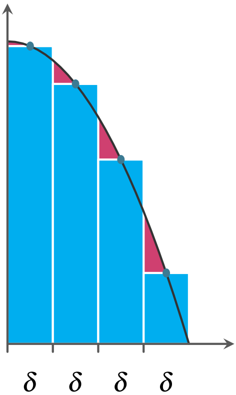
Derivative (rate of change) of Area (Integral) under curve, is height of curve.

Calculus



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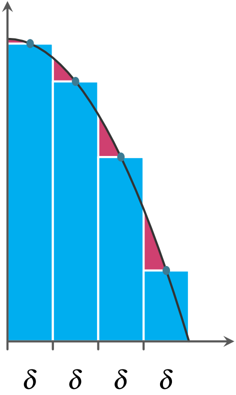
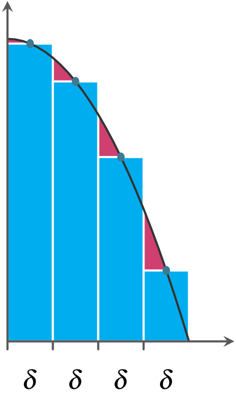


Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

Calculus



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"Area is defined as rectangles and add up some thin ones."

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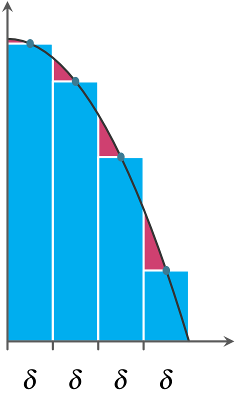

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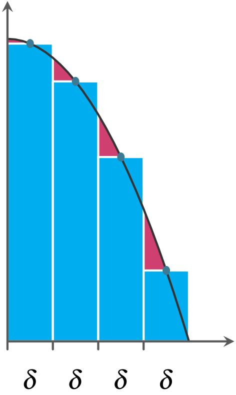

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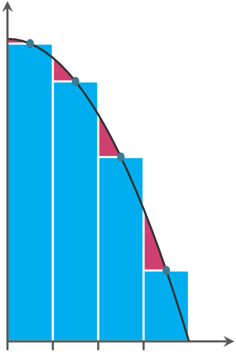

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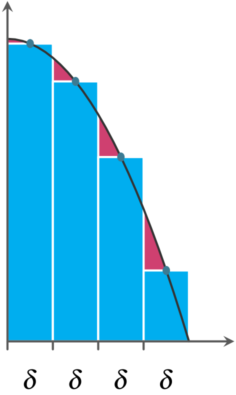

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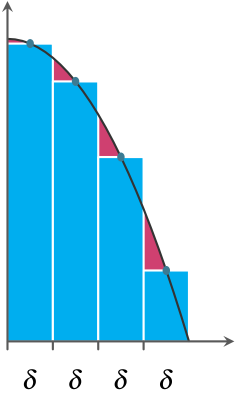

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"Area ($F(\cdot)$) under $f(x)$ grows at x, $F'(x)$, by $f(x)$"
Thus $F'(x) = f(x)$.

Arguments, reasoning.

What you know: slope, limit.

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yields calculus.

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Knowing how to program

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Knowing how to program plus some syntax (google) gives the ability to program.

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...plus reasoning.

CS 70 : ideas.

Induction

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Induction $\equiv$ every integer has a next one.

CS 70 : ideas.

Induction $\equiv$ every integer has a next one. Graph theory.

- Number of edges is sum of degrees.
- $\Delta + 1$ coloring. Neighbors only take up Δ.
- Connectivity plus connected components.
- Eulerian paths: if you enter you can leave.
- Euler's formula: tree has $v - 1$ edges and 1 face plus
  - each extra edge makes additional face.
  - $v - 1 + (f - 1) = e$

CS 70 : ideas.

Number theory.
A divisor of x and y divides $x - y$.
The remainder is always smaller than the divisor.
$\implies$ Euclid's GCD algorithm.
Multiplicative Inverse.
Fermat's theorem from function with inverse is a bijection.
Gives RSA.

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Error Correction.
(Any) Two points determine a line.
(well, and d points determine a degree $d+1$-polynomials.
Cuz, factoring.
Find line by linear equations.
If a couple are wrong, then multiply them by zero, i.e., Error polynomial.

CS70 and your future?

What's going on?

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Define. Understand properties. And build from there.

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....and you will pursue probability in this course.