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More detail: $even+even-even = 2q+2k-2m = 2(q+k-m)$.

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$$a - b + c = 11(k - 9a - b)$$

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$$
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That is 11|alternating sum of digits. □

CS70: Note 3. Induction!

Poll. What's the biggest number?

(A) 100

(B) 101

(C) n+1

(D) infinity.

(E) This is about the "recursive leap of faith."

The natural numbers.

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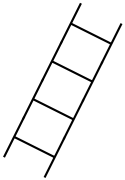
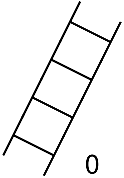


The natural numbers.

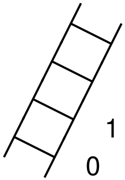
0,



0

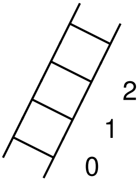
The natural numbers.

0, 1,



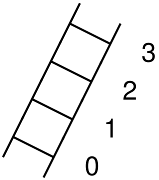
The natural numbers.

0, 1, 2,



The natural numbers.

0, 1, 2, 3,



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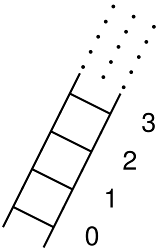

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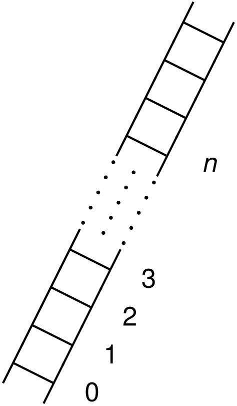

0, 1, 2, 3,
…, n,

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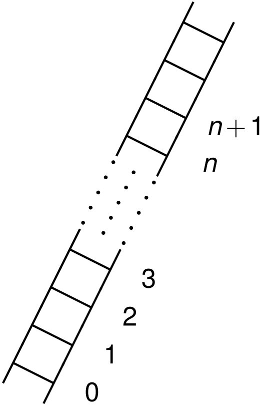

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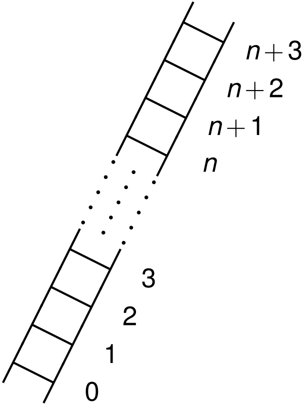

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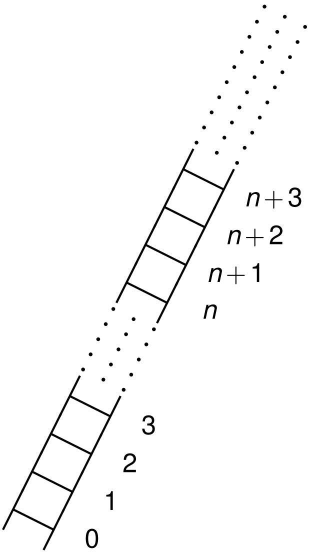

$$0, 1, 2, 3, \quad \ldots, n, n+1, n+2, n+3, \ \ldots$$

A formula.

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- Prove $P(k+1)$. "Induction Step."
- $\implies P(n)$ is true for all $n \in \mathbb{N}$.

Gauss induction proof.

Theorem: For all natural numbers n, $0+1+2\cdots n = \frac{n(n+1)}{2}$

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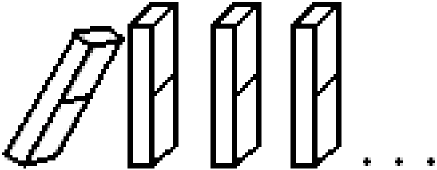
Notes visualization

Note's visualization: an infinite sequence of dominos.



Prove they all fall down;

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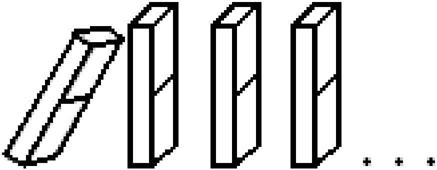

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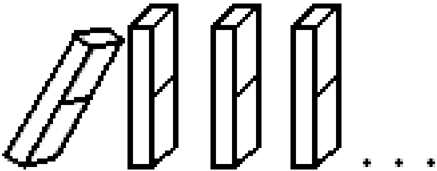

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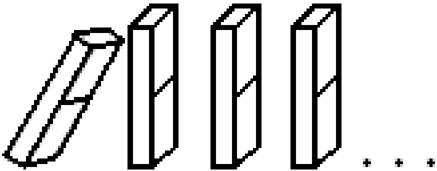

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  "kth domino falls implies that $k+1$st domino falls"

Climb an infinite ladder?

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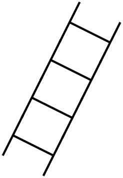
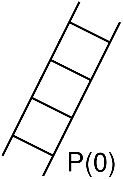


Climb an infinite ladder?

$P(0)$



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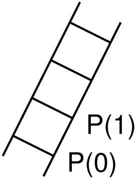

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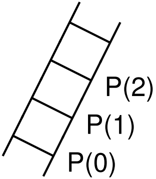

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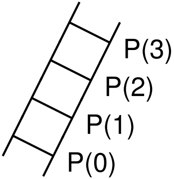
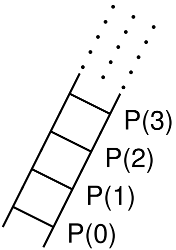


Climb an infinite ladder?



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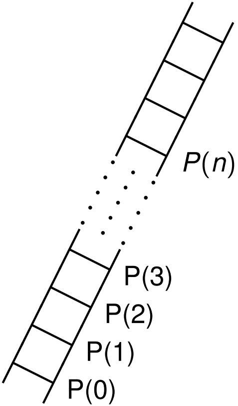

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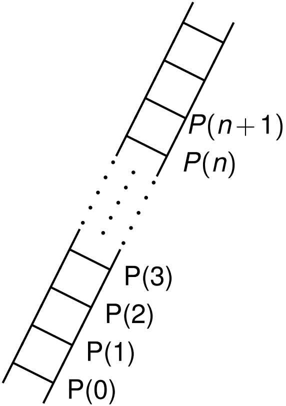

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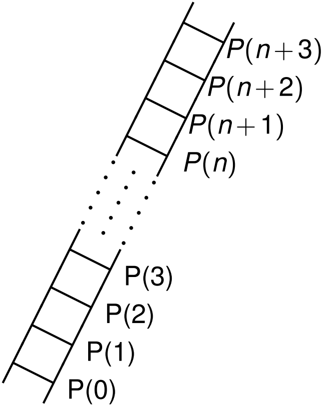

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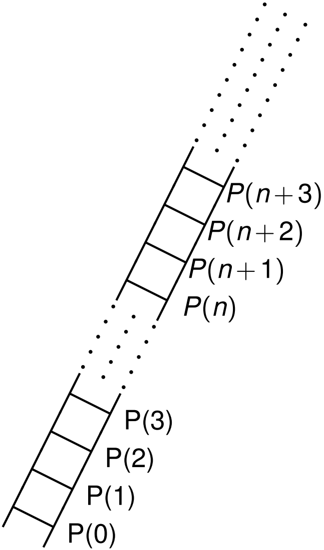

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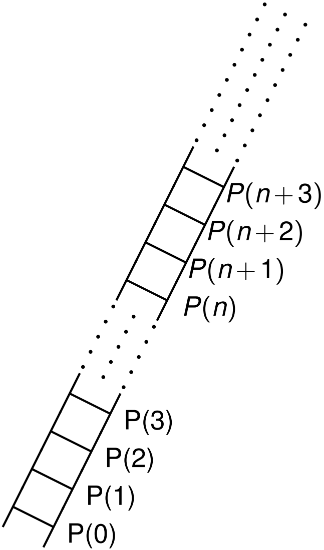

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Your favorite example of forever..

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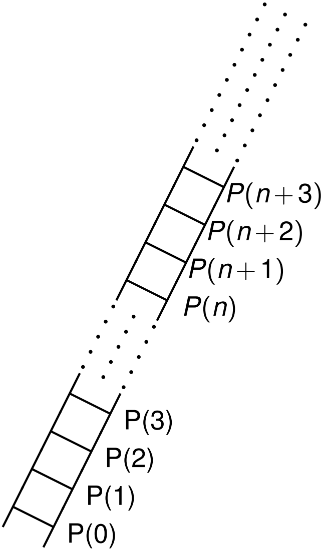

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Your favorite example of forever..or the natural numbers...

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=0}^{n} i = \frac{n(n+1)}{2})$

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Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Poll: What did Gauss use in the proof?

(A) Every natural number has a next number.

(B) The recursive leap of faith.

(C) $2^k > k$.

(D) $\forall k \in \mathbb{N}, \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

Another Induction Proof.

Theorem: For every $n \in N$, $n^3 - n$ is divisible by 3. $(3|(n^3 - n)$).

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$(k+1)^3 - (k+1)$

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Theorem: For every $n \in N$, $n^3 - n$ is divisible by 3. $(3|(n^3 - n))$.

Proof: By induction.
Base Case: $P(0)$ is "$(0^3) - 0$" is divisible by 3. Yes!
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We used everything above except (A) and (E), cuz is false.

Two color theorem: example.

Any map formed by dividing the plane into regions by drawing straight lines can be properly colored with two colors.

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Proper coloring: for each line segment the regions on the two sides have different colors.1

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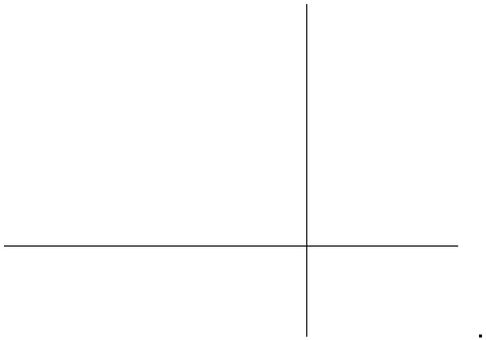

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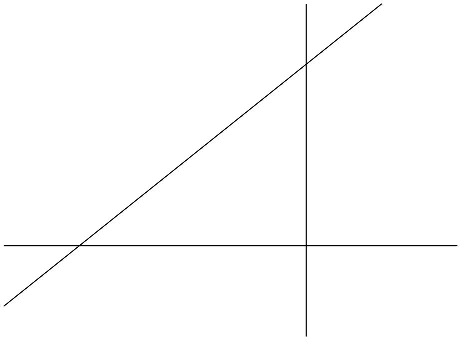

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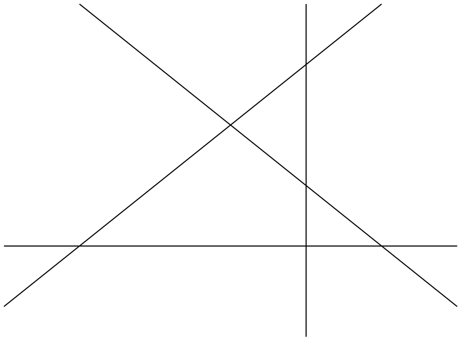

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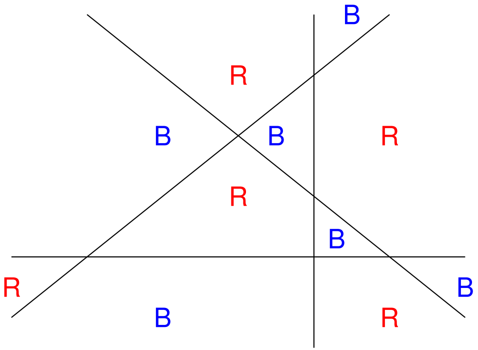

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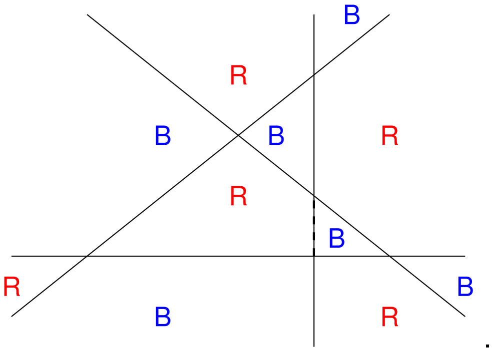

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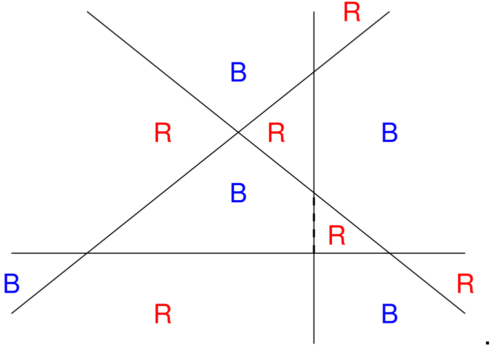

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Two color theorem: proof illustration.

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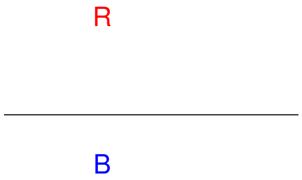

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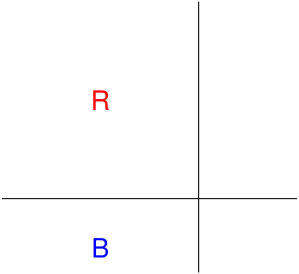

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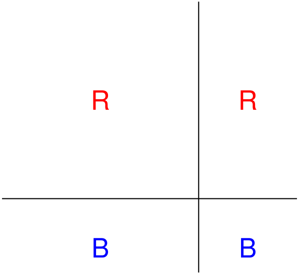

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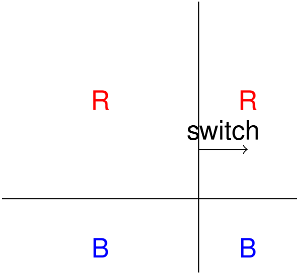

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(Fixes conflicts along new line, and makes no new ones along previous line.)

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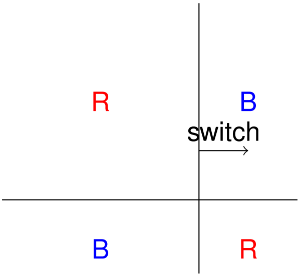

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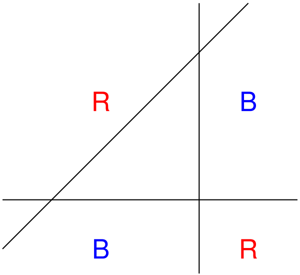

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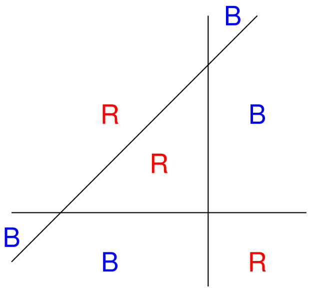

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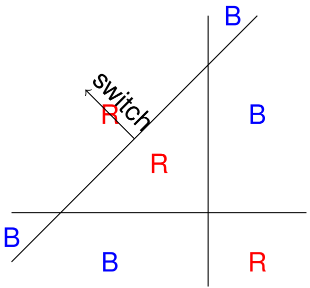

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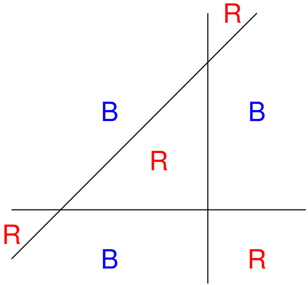

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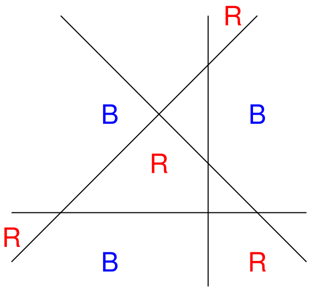

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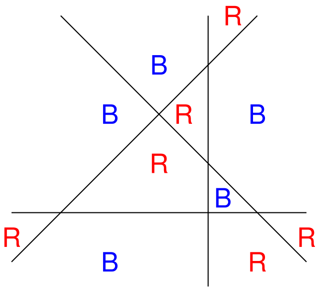

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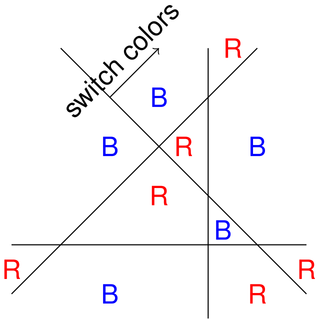

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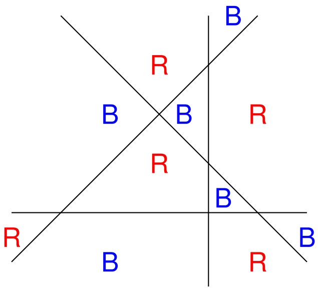

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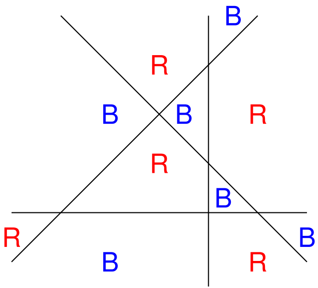

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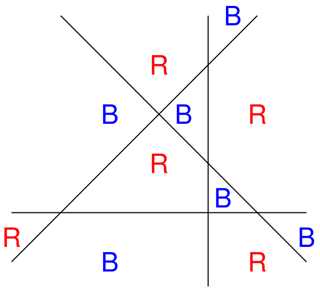

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Poll: what did we use in the proof.

(A) Switching a 2-coloring is a valid coloring.

(B) Definition of 2-coloring.

(C) Definition of adjacent.

(D) Definition of region.

(E) The four color theorem.

Strenthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.

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Base Case: 1 (first odd number) is 1^2.

Induction Hypothesis: Sum of first k odds is perfect square $a^2 = k^2$.

Induction Step:

1. The $(k+1)$st odd number is $2k+1$.
2. Sum of the first $k+1$ odds is
   $a^2 + 2k + 1 = k^2 + 2k + 1$
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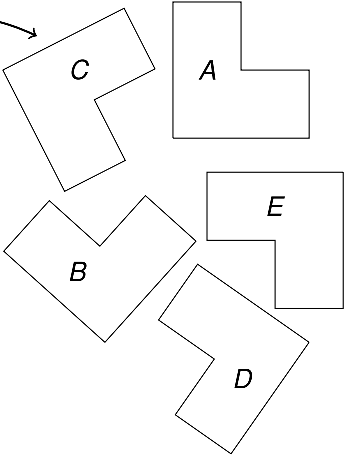
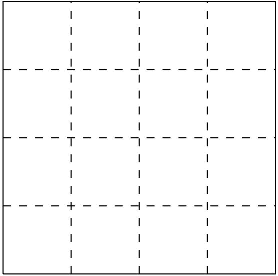
Tiling Cory Hall Courtyard.

Use these L-tiles.



To Tile this 4×4 courtyard.



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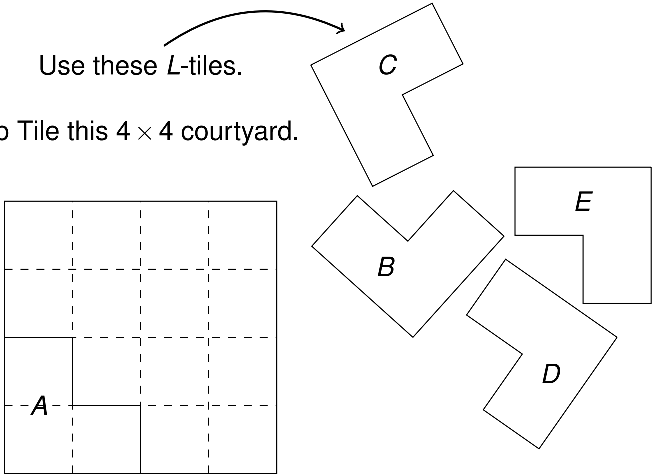

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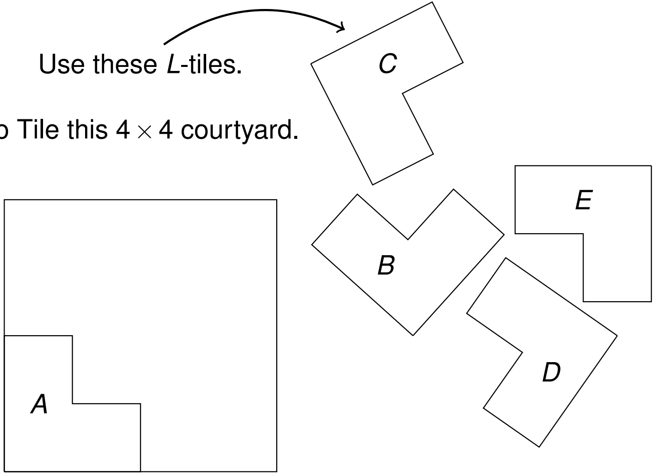

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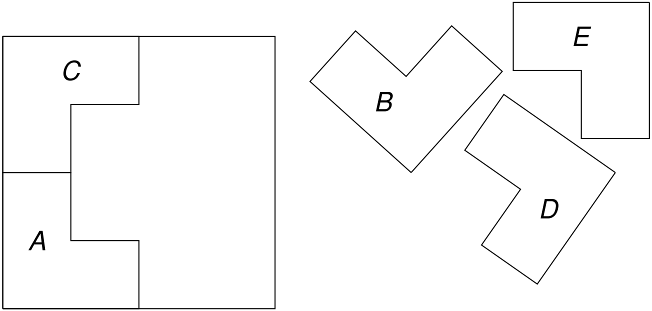

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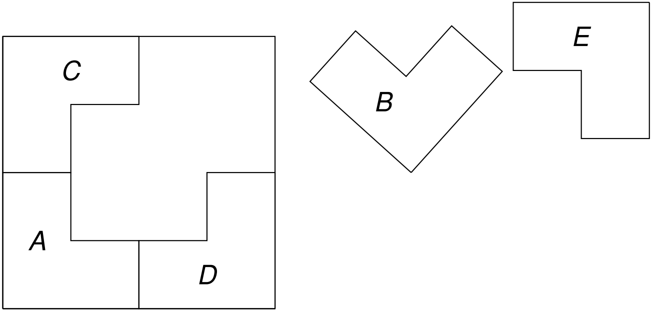

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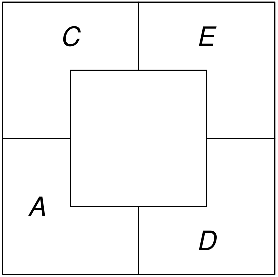

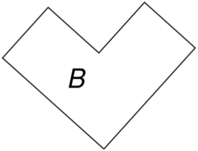

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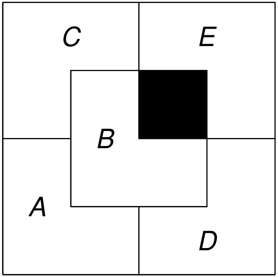

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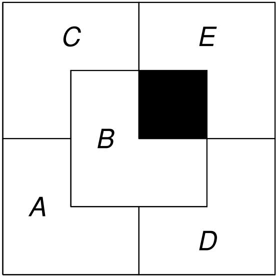

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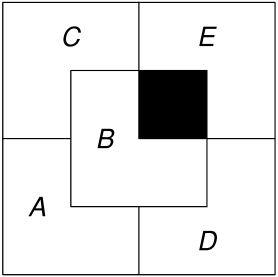

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Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

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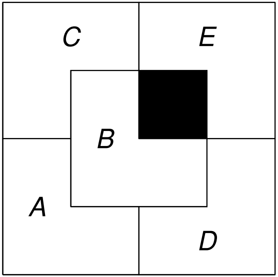

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Tiled 4×4 square with 2×2 L-tiles.

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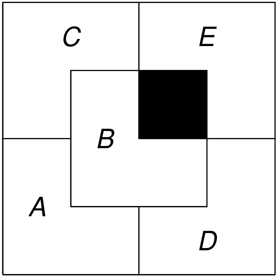

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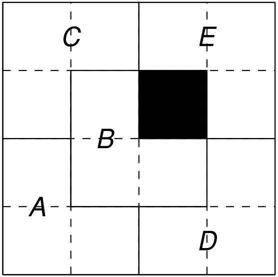

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Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole in center?

Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.
Proof:

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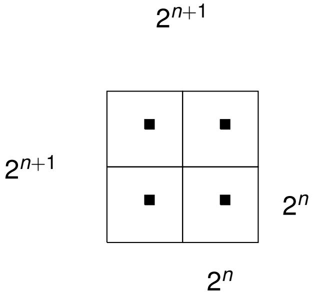

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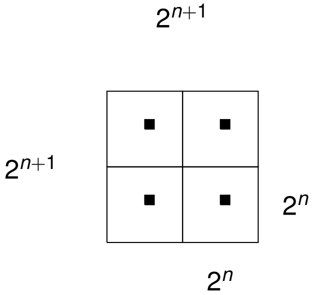

What to do now???

Hole can be anywhere!

Theorem: Can tile the $2^n \times 2^n$ to leave a hole adjacent *anywhere.*

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Better theorem

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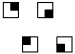
Flipping the orientation can leave hole anywhere.



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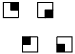
Flipping the orientation can leave hole anywhere. 

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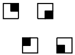

Induction Hypothesis:
“Any $2^n \times 2^n$ square can be tiled with a hole **anywhere.**”
Consider $2^{n+1} \times 2^{n+1}$ square.

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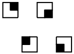

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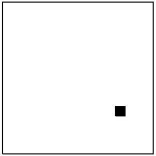

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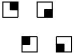

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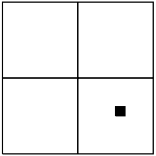

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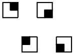

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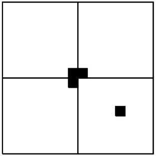

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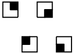

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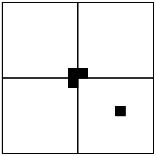

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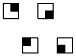

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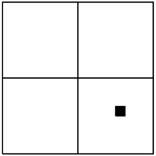

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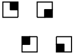
Flipping the orientation can leave hole anywhere. 

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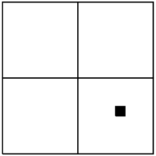

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$P(n) =$ "n can be written as a product of primes. "
Either $n+1$ is a prime or $n+1 = a \cdot b$ where $1 < a, b < n+1$.
$P(n)$ says nothing about a, b!

Strong Induction Principle: If $P(0)$ and

$$(\forall k \in N)((P(0) \wedge \ldots \wedge P(k)) \implies P(k+1)),$$

then $(\forall k \in N)(P(k))$.

$$P(0) \implies P(1) \implies P(2) \implies P(3) \implies \cdots$$

Strong induction hypothesis: "a and b are products of primes"

$\implies$ "$n+1 = a \cdot b =$ (factorization of a)(factorization of b)"

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Theorem: Every natural number $n > 1$ can be written as a (possibly trivial) product of primes.
Definition: A prime n has exactly 2 factors 1 and n.
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Fix to last lecture.

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A number that is not prime is divisible by another number.

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Prime p divides n by principle of strong induction. □

Well Ordering Principle and Induction.

If $(\forall n)P(n)$ is not true, then $(\exists n)\neg P(n)$.

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Note: can do with different definition of smallest.
For example. Use reduced form: a/b and order by $a+b$.

Well ordering principle.

Thm: All natural numbers are interesting.

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0 is interesting...

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Thus: All natural numbers are interesting.

Tournaments have short cycles

Def: A **round robin tournament on** n **players**: every player p plays every other player q, and either $p \to q$ (p beats q) or $q \to p$ (q beats p.)

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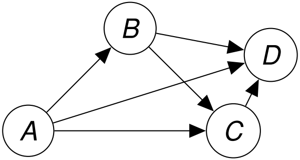

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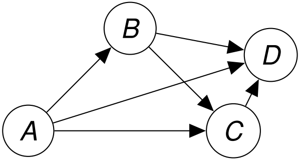

Theorem: Any tournament that has a cycle has a cycle of length 3.

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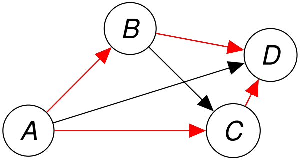

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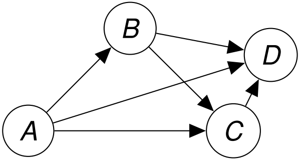

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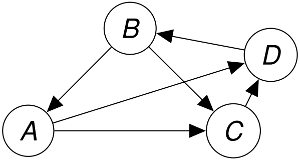

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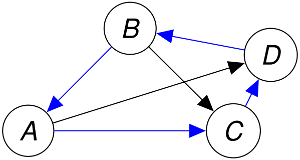

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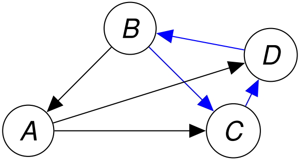

Theorem: Any tournament that has a cycle has a cycle of length 3.

Tournament has a cycle of length 3 if at all.

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Assume the the **smallest cycle** is of length k.

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Case 1: Of length 3.

Tournament has a cycle of length 3 if at all.

Assume the the **smallest cycle** is of length k.

Case 1: Of length 3. Done.

Tournament has a cycle of length 3 if at all.

Assume the the **smallest cycle** is of length k.

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Case 2: Of length larger than 3.

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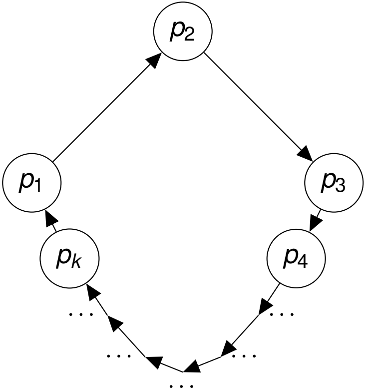

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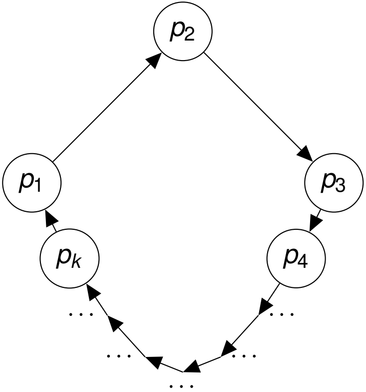

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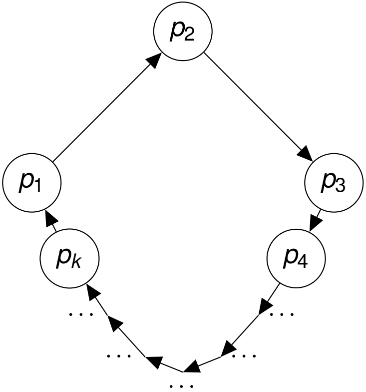

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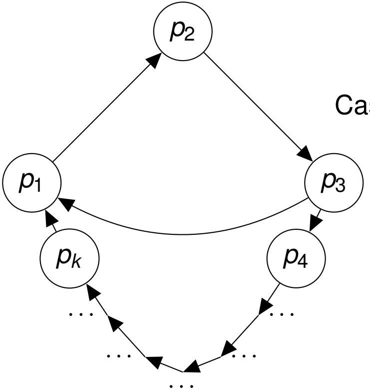

Case 2a: "$p_3 \rightarrow p_1$"

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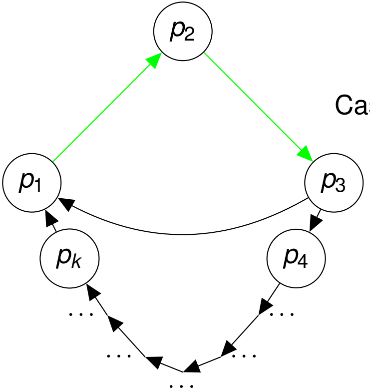

Case 2a: "$p_3 \to p_1$" $\Longrightarrow$ 3 cycle

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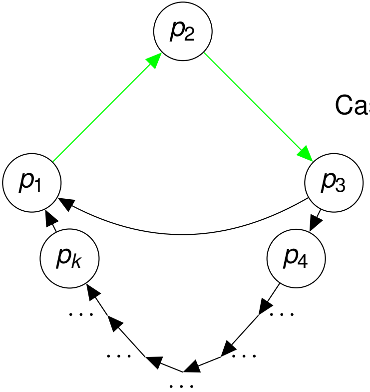

Case 2a: "$p_3 \to p_1$" $\implies$ 3 cycle

Contradiction.

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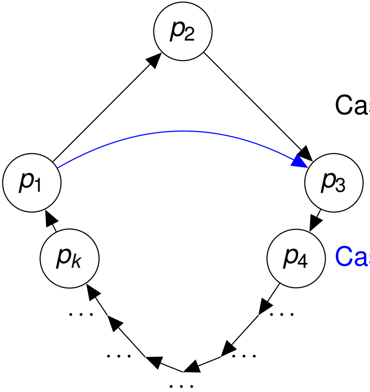

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Case 2b: "$p_1 \rightarrow p_3$"

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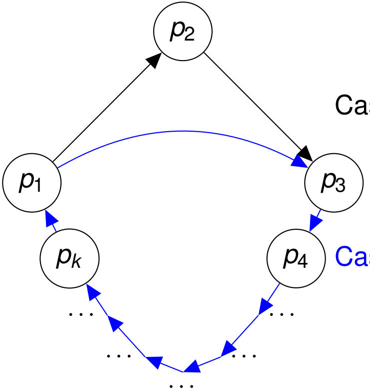

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Case 2b: "$p_1 \to p_3$" $\Longrightarrow$ $k-1$ length cycle!

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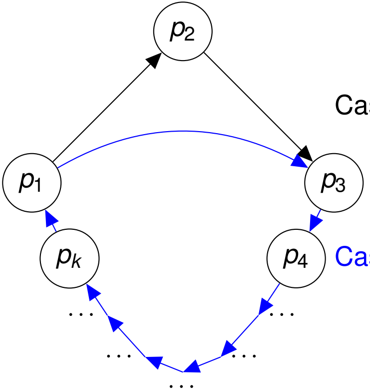

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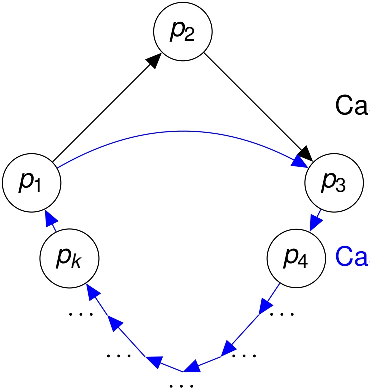

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□

Horses of the same color...

Theorem: All horses have the same color.

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Base Case: $P(1)$ - trivially true.

Horses of the same color...

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Induction Hypothesis: $P(k)$ - Any k horses have the same color.

Horses of the same color...

Theorem: All horses have the same color.

Base Case: $P(1)$ - trivially true.

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All k must have the same color.	$1, 2, 3, \ldots, k, k+1$

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Horses of the same color...

Theorem: All horses have the same color.

Base Case: $P(1)$ - trivially true.
New Base Case: $P(2)$: there are two horses with same color.

Induction Hypothesis: $P(k)$ - Any k horses have the same color.

Induction step $P(k+1)$?
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A horse in the middle in common!

Fix base case.

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More subtle to catch errors in proofs of correct theorems!!

Sad Islanders...

Island with 100 possibly blue-eyed and green-eyed inhabitants.

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First rule of island: Don’t talk about eye color!

Visitor: “I see someone has green eyes.”

Result: What happens?
(A) Nothing, no information was added.
(B) Information was added, maybe?
(C) They all leave the island.
(D) They all leave the island on day 100.

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On day 100,

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On day 100, they all leave.

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Why?

They know induction.

Thm: If there are n villagers with green eyes they leave on day n.

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If n people with green eyes, they would leave on day n.

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Base: $n = 1$. Person with green eyes leaves on day 1.

Induction hypothesis:
If n people with green eyes, they would leave on day n.

Induction step:
On day $n+1$, a green eyed person sees n people with green eyes.

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Proof:
Base: $n = 1$. Person with green eyes leaves on day 1.

Induction hypothesis:
If n people with green eyes, they would leave on day n.

Induction step:
On day $n+1$, a green eyed person sees n people with green eyes.

But they didn't leave.

They know induction.

Thm: If there are n villagers with green eyes they leave on day n.

Proof:
Base: $n = 1$. Person with green eyes leaves on day 1.

Induction hypothesis:
If n people with green eyes, they would leave on day n.

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On day $n+1$, a green eyed person sees n people with green eyes.

But they didn't leave.

So there must be $n+1$ people with green eyes.

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Wait! Visitor added no information.

Common Knowledge.

Using knowledge about what other people's knowledge (your eye color) is.

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On day 1, everyone knows everyone sees more than zero.

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On day 99, everyone knows no one sees 98

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On day 99, everyone knows no one sees 98
since everyone knows everyone else does not see 97...

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On day 100, ...uh oh!

Another example:

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Using knowledge about what other people's knowledge (your eye color) is.

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Another example:
Emperor's new clothes!

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On day 99, everyone knows no one sees 98
since everyone knows everyone else does not see 97...

On day 100, ...uh oh!

Another example:
Emperor's new clothes!
No one knows other people see that he has no clothes.

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Using knowledge about what other people's knowledge (your eye color) is.

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On day 2, everyone knows everyone sees more than one.

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On day 99, everyone knows no one sees 98
 since everyone knows everyone else does not see 97...

On day 100, ...uh oh!

Another example:
Emperor's new clothes!
 No one knows other people see that he has no clothes.
 Until kid points it out.

Summary: principle of induction.

Today: More induction.

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Today: More induction.

$(P(0)$

Summary: principle of induction.

Today: More induction.

$(P(0) \land ((\forall k \in N)(P(k) \implies P(k+1))))$

Summary: principle of induction.

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$$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$$

Summary: principle of induction.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0

Summary: principle of induction.

Today: More induction.

$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.

Summary: principle of induction.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.
Ind. Step: Prove.

Summary: principle of induction.

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$$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$$

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Base Case: Prove $P(n_0)$.
Ind. Step: Prove. For all values, $n \geq n_0$,

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Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.
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Base Case: Prove $P(n_0)$.
Ind. Step: Prove. For all values, $n \geq n_0$, $P(n) \implies P(n+1)$.
Statement is proven!

Summary: principle of induction.

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Strong Induction:

Summary: principle of induction.

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Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.
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Strong Induction:
$(P(0) \wedge ((\forall n \in N)(P(n) \implies P(n+1)))) \implies (\forall n \in N)(P(n))$

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Strong Induction:
$(P(0) \land ((\forall n \in N)(P(n) \implies P(n+1)))) \implies (\forall n \in N)(P(n))$

Also Today: strengthened induction hypothesis.

Summary: principle of induction.

Today: More induction.

$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.
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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.

Summary: principle of induction.

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.

Summary: principle of induction.

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.

Summary: principle of induction.

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.
Not same as strong induction.

Summary: principle of induction.

Today: More induction.

$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

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Statement is proven!

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.
Not same as strong induction. E.g., used in product of primes proof.

Summary: principle of induction.

Today: More induction.

$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.
Ind. Step: Prove. For all values, $n \geq n_0$, $P(n) \implies P(n+1)$.
Statement is proven!

Strong Induction:
$(P(0) \wedge ((\forall n \in N)(P(n) \implies P(n+1)))) \implies (\forall n \in N)(P(n))$

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Not same as strong induction. E.g., used in product of primes proof.

Induction $\equiv$ Recursion.

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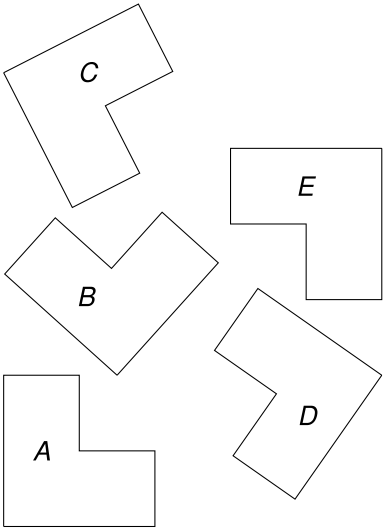

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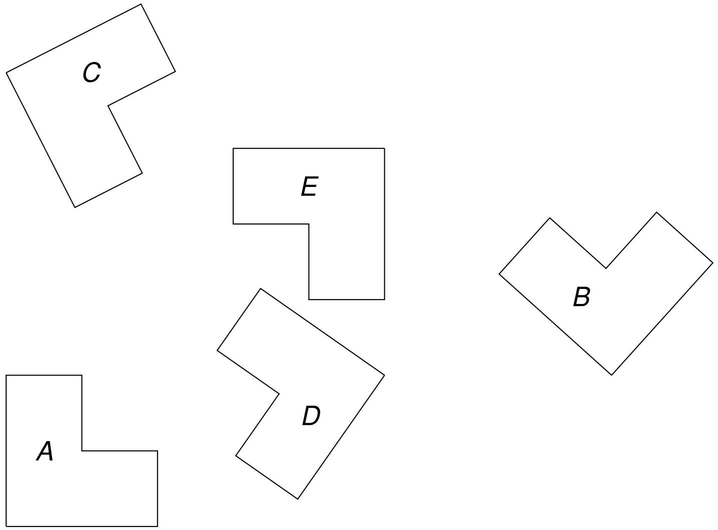

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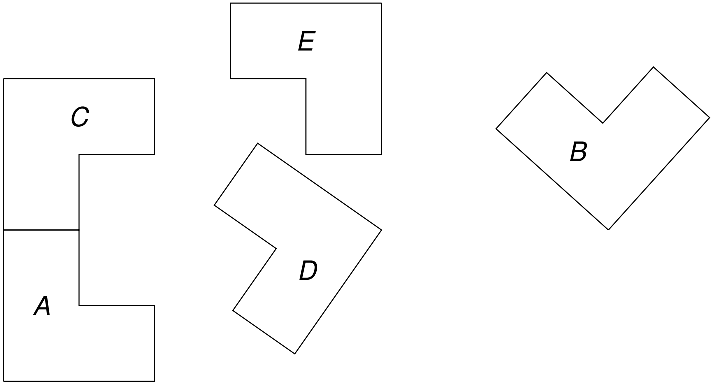

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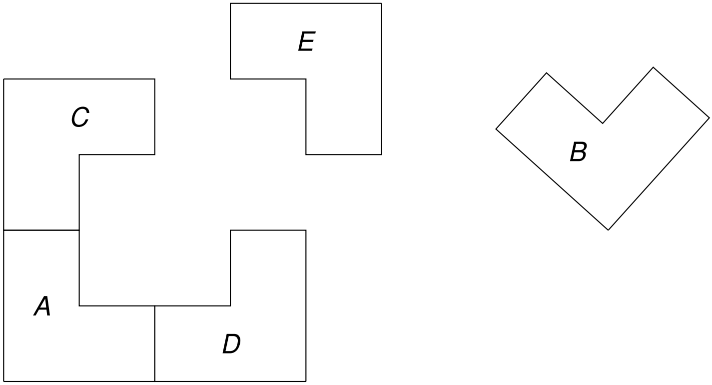

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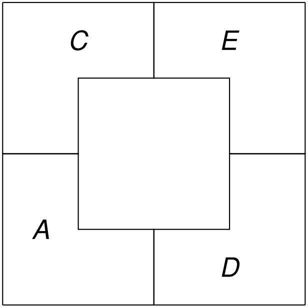

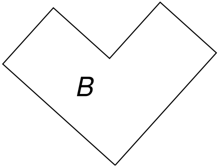

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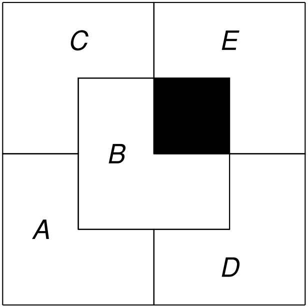

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