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In S, (g, b^*) is pair.

g prefers b to b^*.

T is job optimal, so b prefers g to its partner in S.

(g, b) is Rogue couple for S

S is not stable.

Contradiction. $\square$

Notes: Not really induction.
Structural statement: Job optimality

How about for candidates?

Theorem: Job Propose and Reject produces candidate-pessimal pairing.

T – pairing produced by JPR.

S – worse stable pairing for candidate g.

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S is not stable.

Contradiction. □

Notes: Not really induction.
Structural statement: Job optimality $\implies$ Candidate pessimality.

Quick Questions.

How does one make it better for candidates?

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Propose and Reject - stable matching algorithm. One side proposes.

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Jobs Propose $\implies$ job optimal.

Candidates propose. $\implies$ optimal for candidates.

Residency Matching..

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The method was used to match residents to hospitals.

Residency Matching..

The method was used to match residents to hospitals.

Hospital optimal....

Residency Matching..

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..until 1990's...

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Another variation: couples.

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Takeaways.

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Proofs carefully use definition:
Stability:
Improvement Lemma plus every day the job gets to choose.
Optimality proof:
Job Optimality:
contradiction of the existence of a better *stable* pairing.
that is, no rogue couple by improvement, job choice, and well ordering principle. Candidate Pessimality:
contradiction plus cuz job optimality implies better pairing.

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Life Lesson: ask,

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Life Lesson: ask, you will do better even if rejection is hard.

Lecture 5: Graphs.

Graphs!

Lecture 5: Graphs.

Graphs!
 Definitions: model.

Lecture 5: Graphs.

Graphs!
- Definitions: model.
- Fact!

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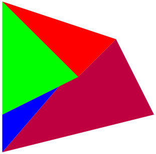
- Definitions: model.
- Fact!

Map Coloring.



Map Coloring.

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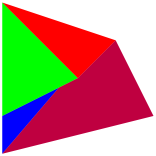

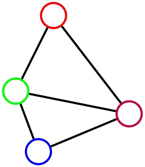

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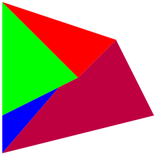
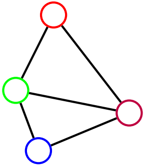
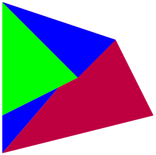
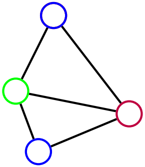




Fewer Colors?

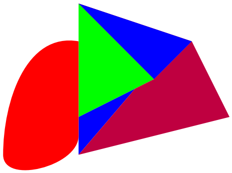
Map Coloring.

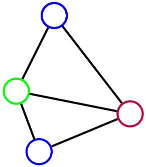




Yes! Three colors.

Map Coloring.





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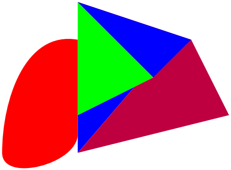 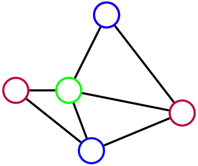

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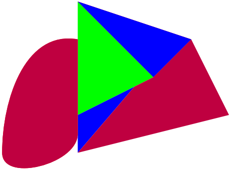

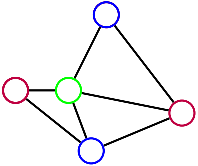

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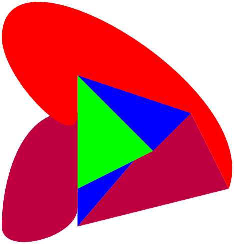

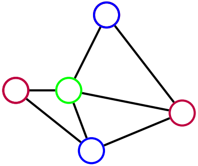

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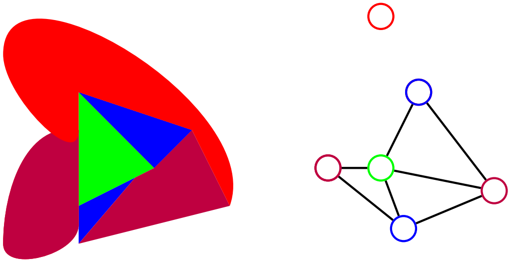

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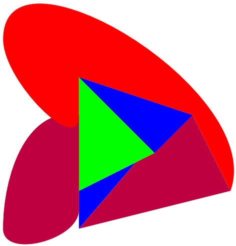

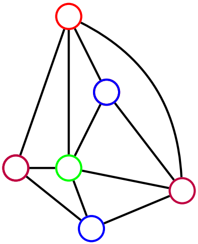

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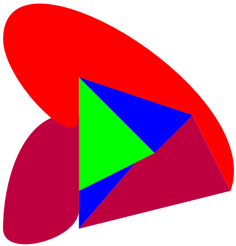
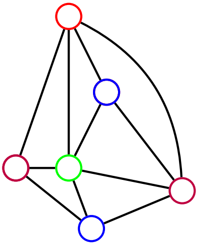
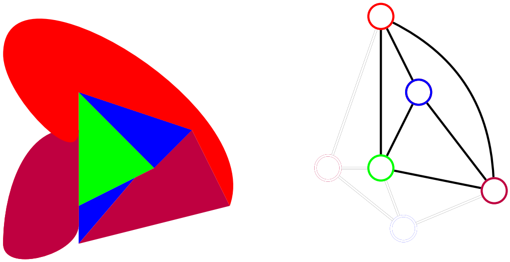


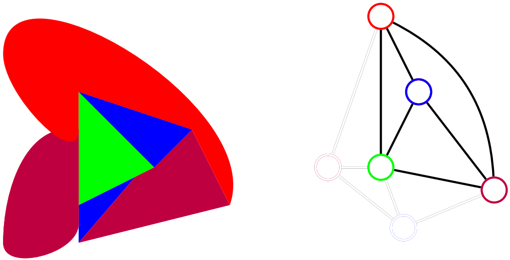


Fewer Colors?

Map Coloring.

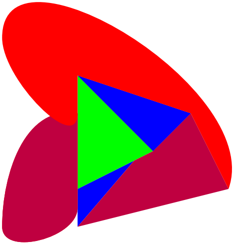


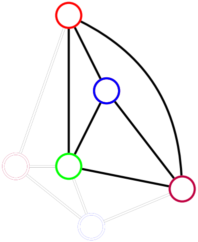
Map Coloring.



Four colors required!

Map Coloring.

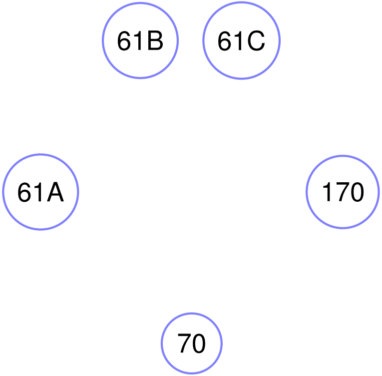




Four colors required!

Theorem: Four colors enough for maps on the plane.

Scheduling: coloring.



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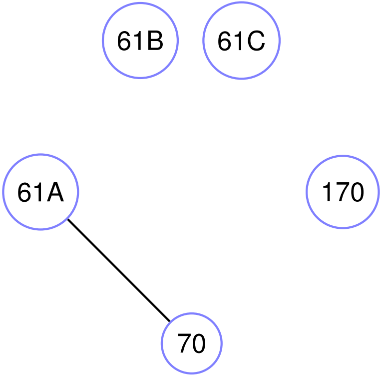

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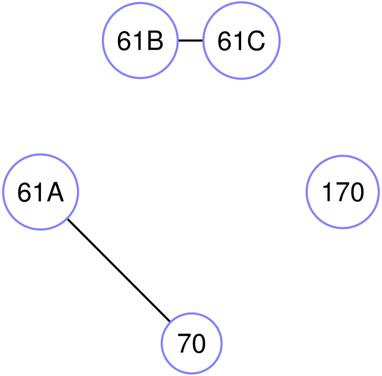

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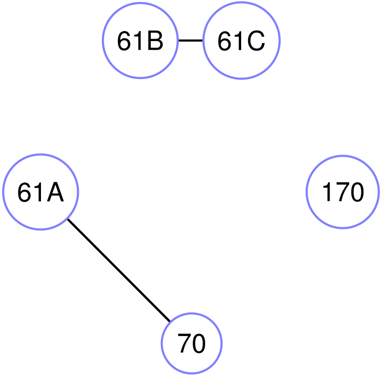

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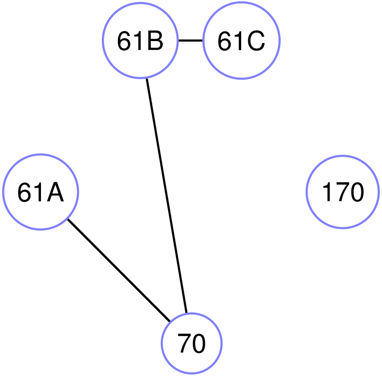

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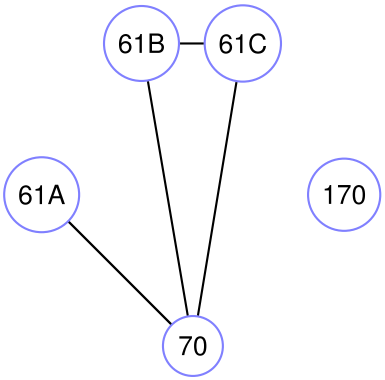
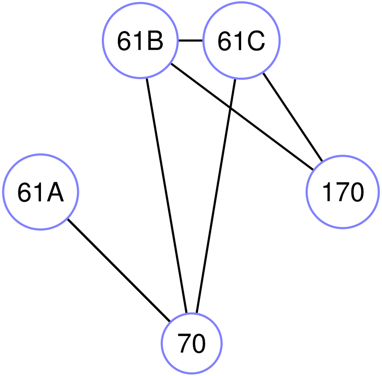
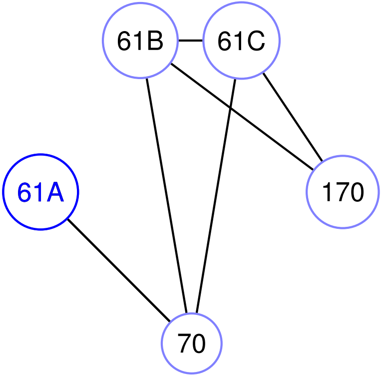


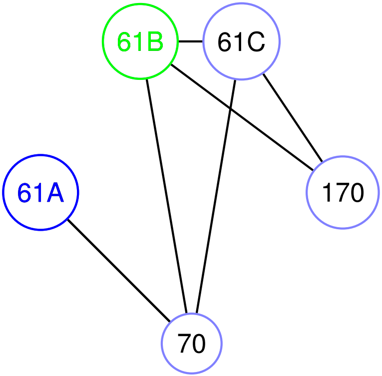
Scheduling: coloring.



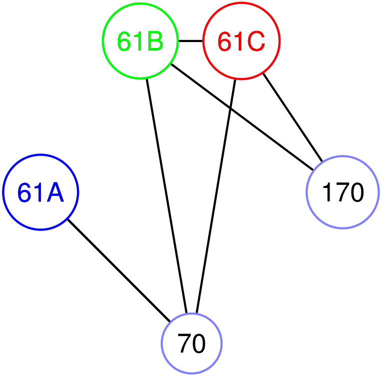
Scheduling: coloring.



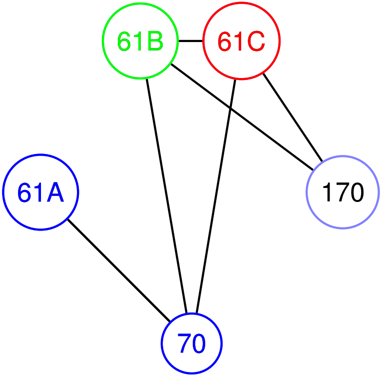
Scheduling: coloring.



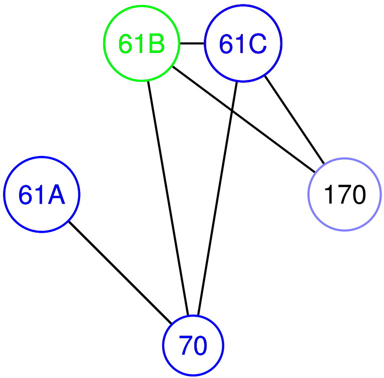
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.



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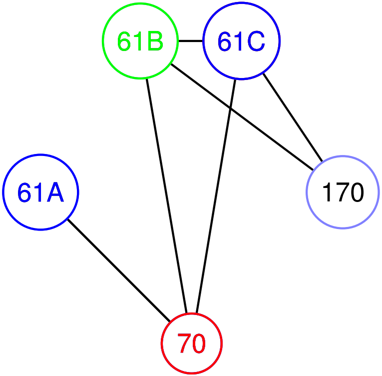

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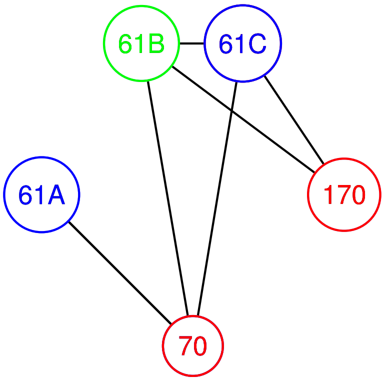

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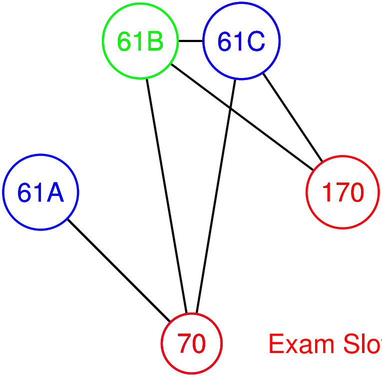
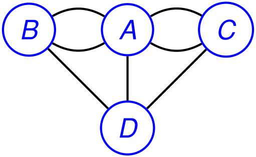


Exam Slot 1.

Exam Slot 2.

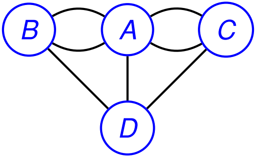
Exam Slot 3.

Graphs: formally.



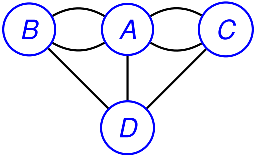
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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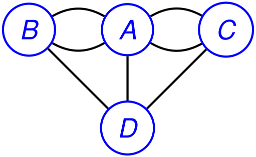
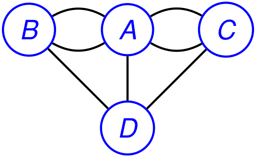


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



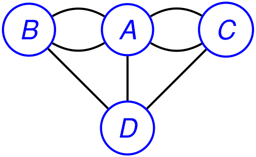
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



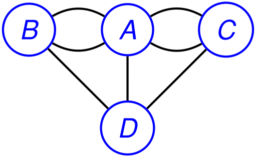
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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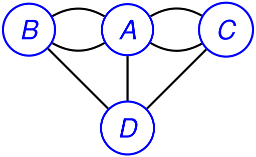
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



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V - set of vertices.

$\{A, B, C, D\}$

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$\{\{A, B\}, \{A, B\}$

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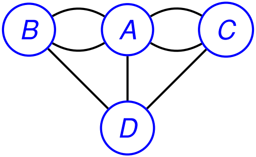

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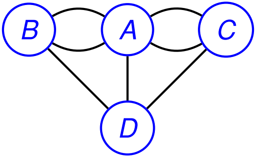
$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



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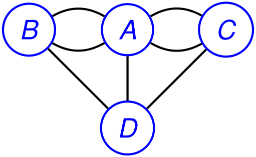
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.

Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

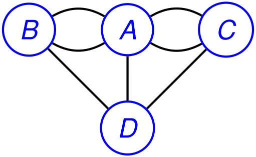
$E \subseteq V \times V$ - set of edges.

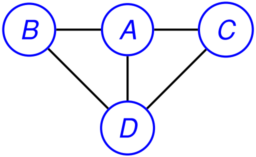
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





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V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

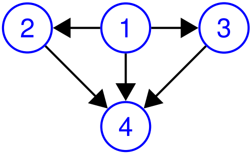
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

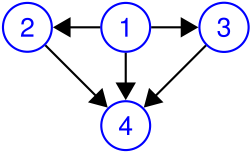
Multigraph above.

Directed Graphs



$G = (V, E)$.

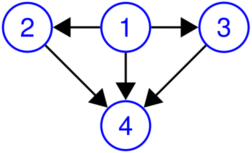
Directed Graphs



$G = (V, E)$.

V - set of vertices.

Directed Graphs

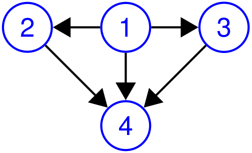


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



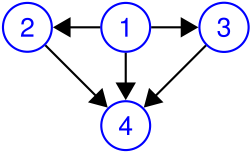
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

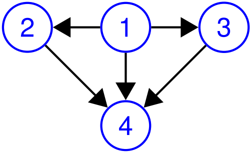
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



$G = (V, E)$.

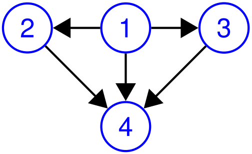
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



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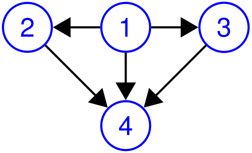
Directed Graphs



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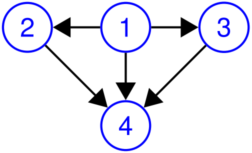
Directed Graphs



$G = (V, E)$.

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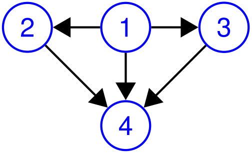
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



$G = (V, E)$.

V - set of vertices.

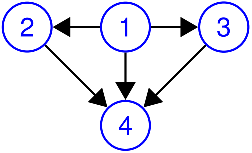
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

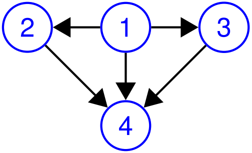
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2,

Directed Graphs



$G = (V, E)$.

V - set of vertices.

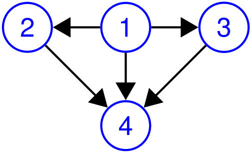
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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V - set of vertices.
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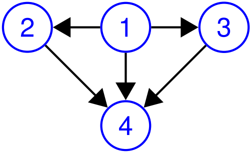
E ordered pairs of vertices.
$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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Tournament: 1 beats 2, ...
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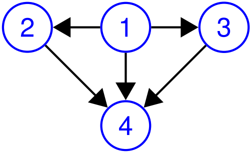
Directed Graphs



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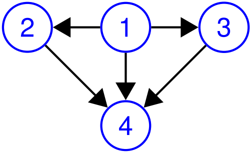
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs

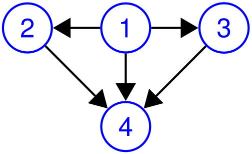


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs



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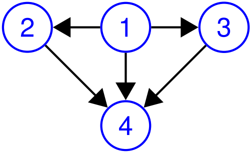
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs



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V - set of vertices.
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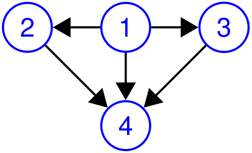
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs



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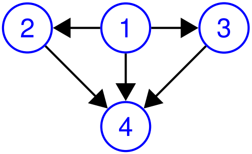
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



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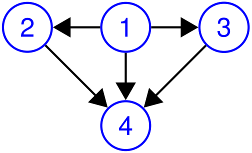
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



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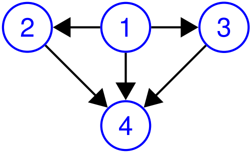
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

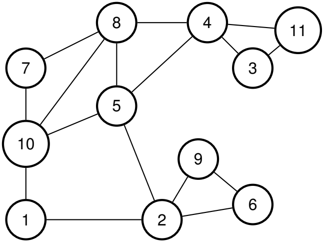
Graph: $G = (V, E)$

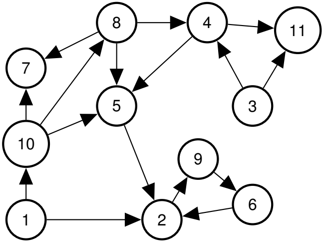
neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



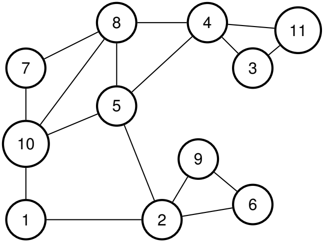


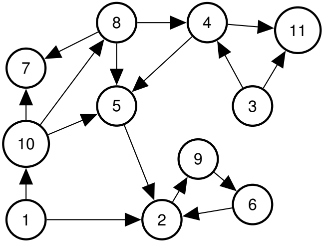
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



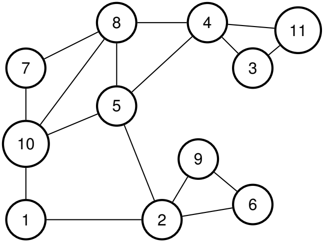


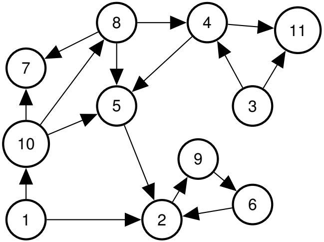
Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



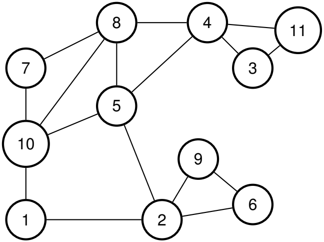


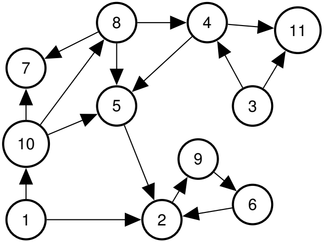
Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



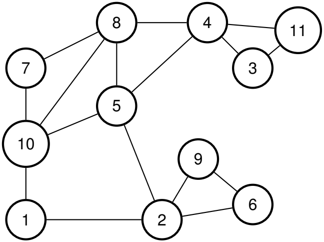


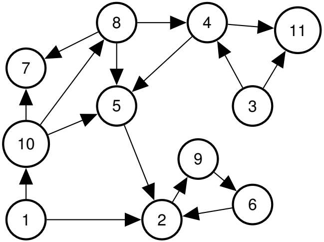
Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



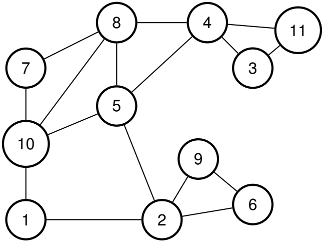


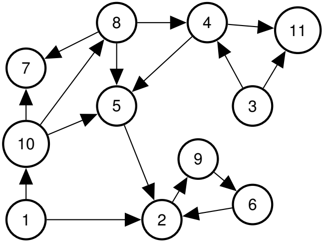
Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





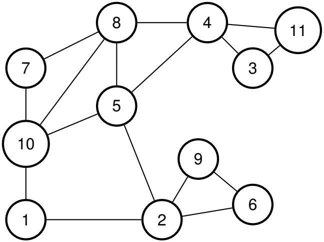
Neighbors of 10? 1,5,7, 8.

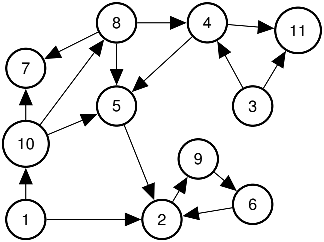
u is neighbor of v if $\{u, v\} \in E$.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

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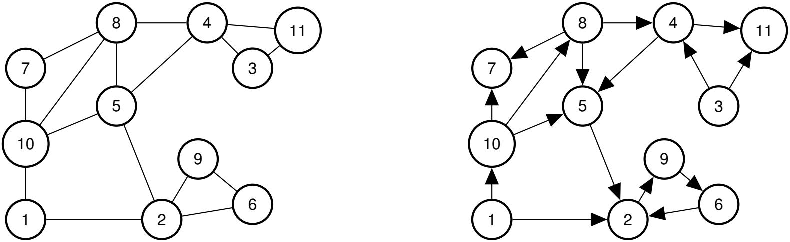
Edge $\{10, 5\}$ is incident to

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



Neighbors of 10? 1,5,7, 8.

u is neighbor of v if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

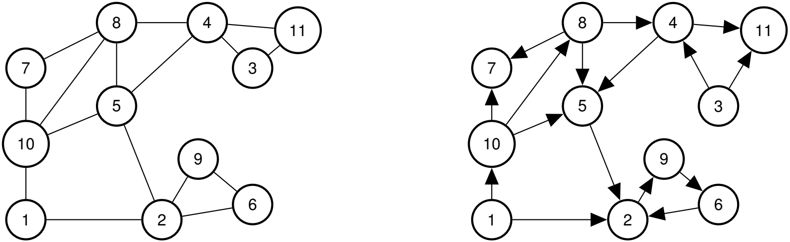
Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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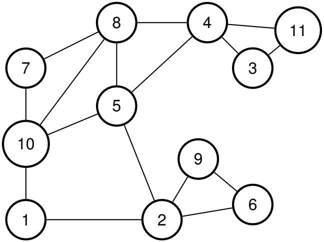
Edge $\{u, v\}$ is incident to u and v.

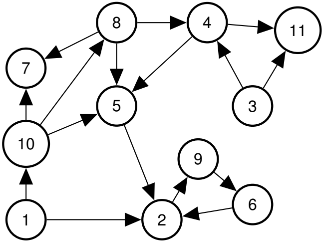
Degree of vertex 1? 2

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Edge $\{u, v\}$ is incident to u and v.

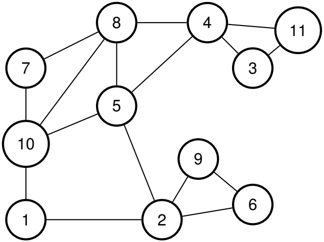
Degree of vertex 1? 2

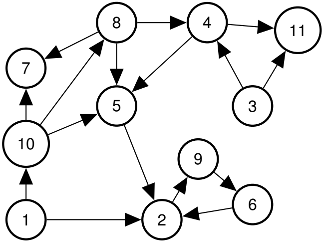
Degree of vertex u is number of incident edges.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1? 2

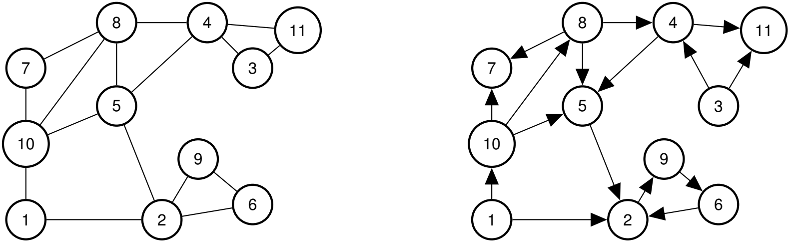
Degree of vertex u is number of incident edges.

Equals number of neighbors in simple graph.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



Neighbors of 10? 1,5,7, 8.

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Edge $\{u, v\}$ is incident to u and v.

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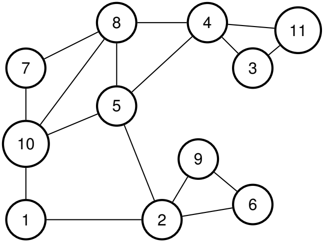
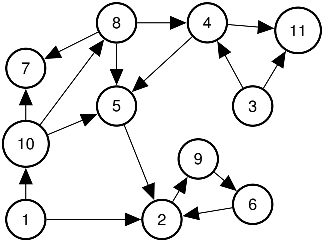
Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

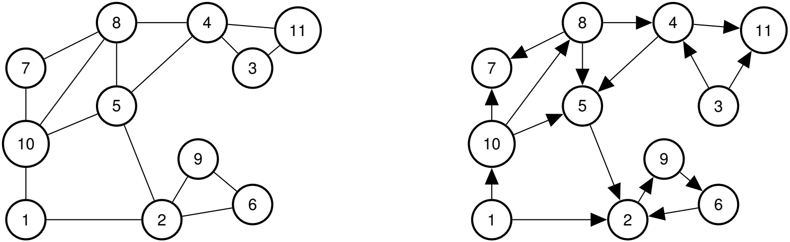
Equals number of neighbors in simple graph.

Directed graph?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



Neighbors of 10? 1,5,7, 8.

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Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

Equals number of neighbors in simple graph.

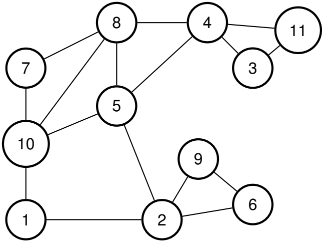
Directed graph?

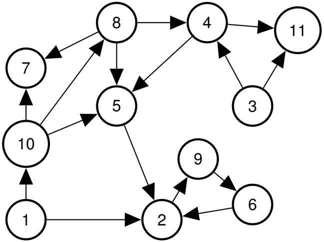
In-degree of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

u is neighbor of v if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

Equals number of neighbors in simple graph.

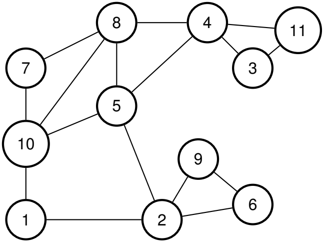
Directed graph?

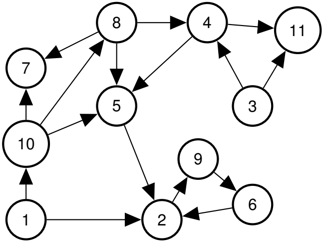
In-degree of 10? 1

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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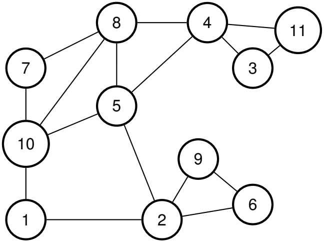

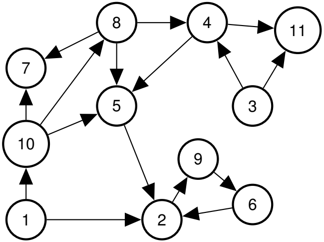

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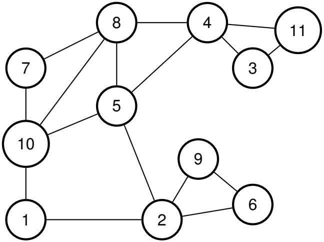

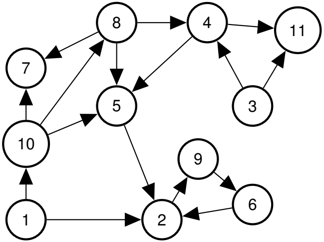

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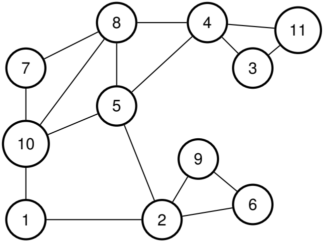

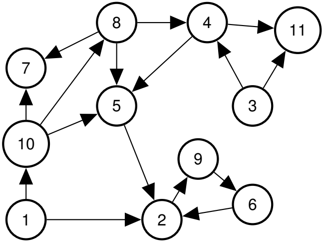

Edge $(8, 5)$ **is incident to:**

(A) Vertex 8.
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(C) Edge $(8, 5)$.
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Graph Concepts and Definitions.

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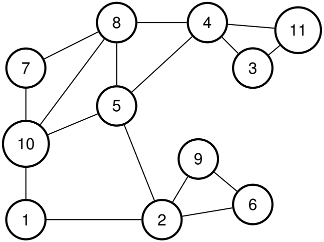

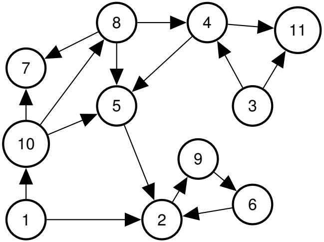

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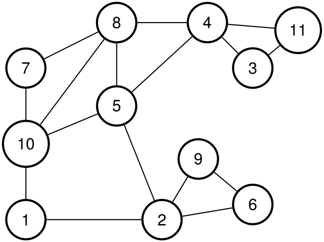

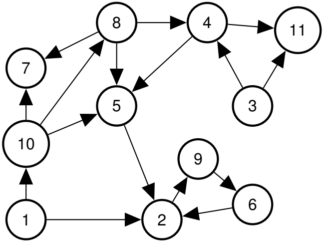

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The degree of a vertex is:

(A) The number of edges incident to it.
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Graph Concepts and Definitions.

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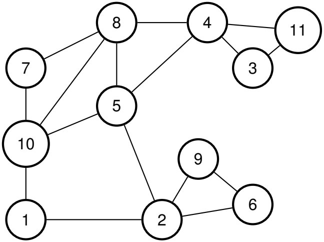

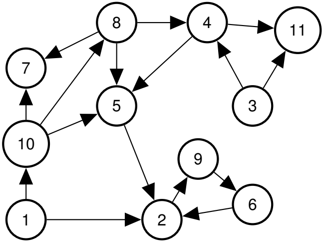

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The sum of the vertex degrees is equal to

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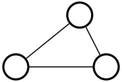
(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)!



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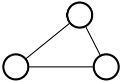

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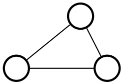

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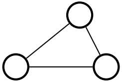

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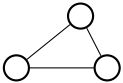

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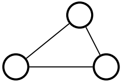

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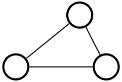

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What? For triangle number of edges is 3, the sum of degrees is 6.

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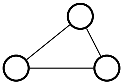

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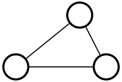

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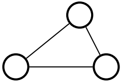

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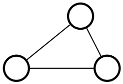

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Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Recall:

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Let's count incidences in two ways.

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What is degree v?

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Let's count incidences in two ways.

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What is degree v? Incidences corresponding to v!

Quick Proof of an Equality.

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

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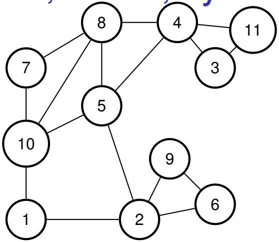
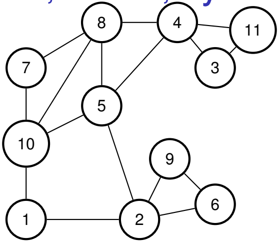
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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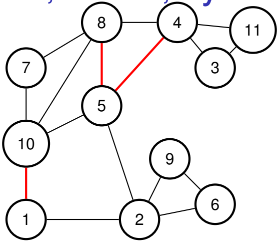

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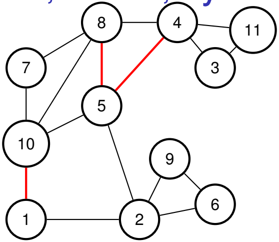
Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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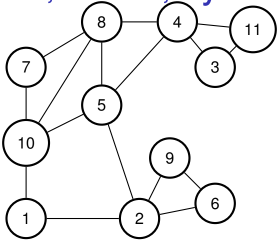

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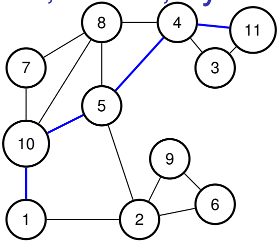
Path?

Paths, walks, cycles, tour.

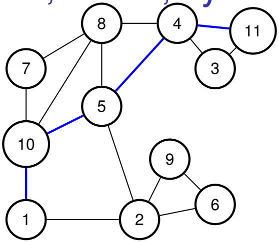


A path in a graph is a sequence of edges.

Path? $\{1,10\}, \{8,5\}, \{4,5\}$? No!

Path? $\{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}$?

Paths, walks, cycles, tour.



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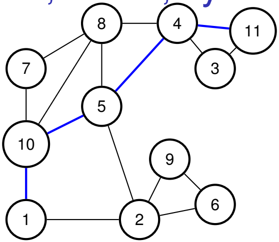

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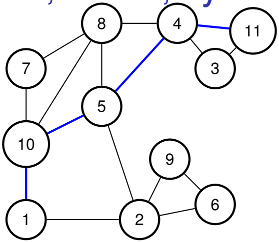

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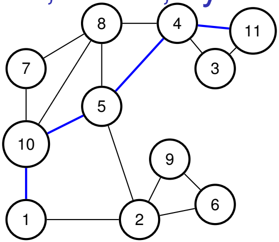
Paths, walks, cycles, tour.



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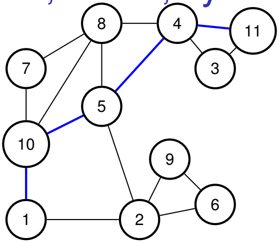
Quick Check! Length of path?

Paths, walks, cycles, tour.



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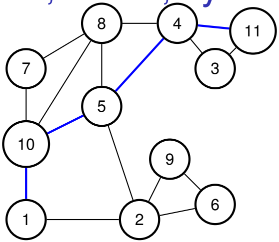
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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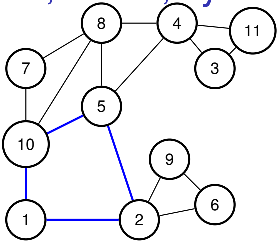
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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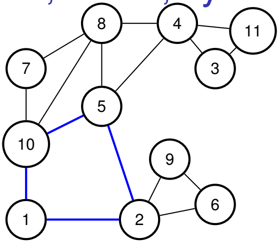
Paths, walks, cycles, tour.



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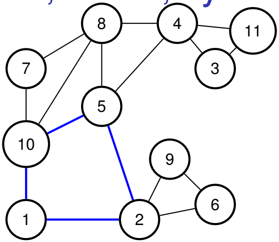

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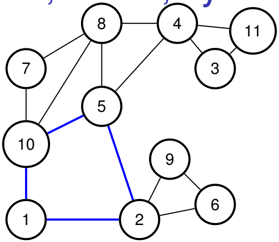

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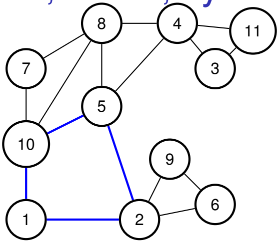
Paths, walks, cycles, tour.



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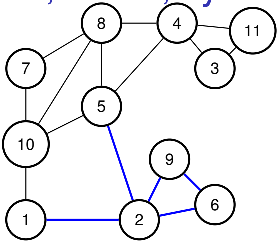

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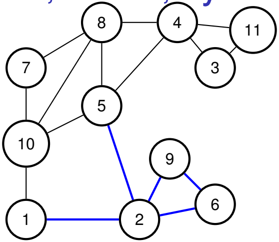

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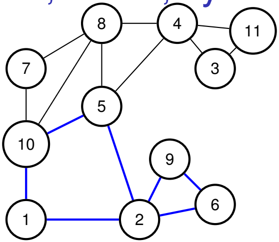

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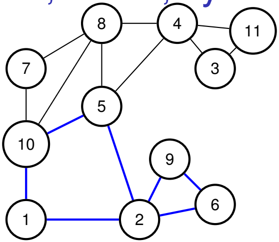

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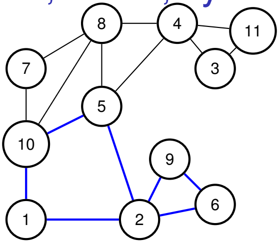

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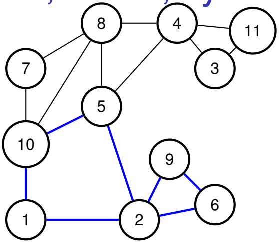

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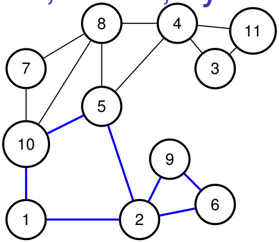
Paths, walks, cycles, tour.



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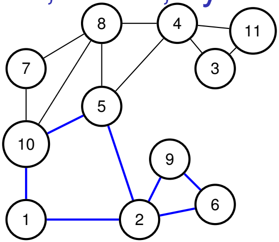

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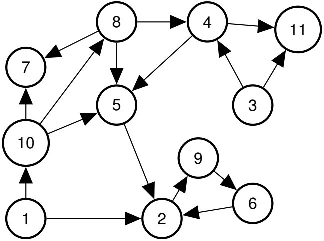
Directed Paths.



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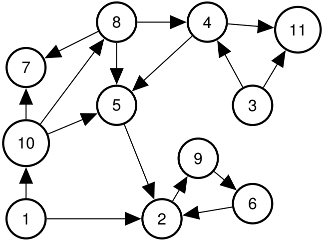

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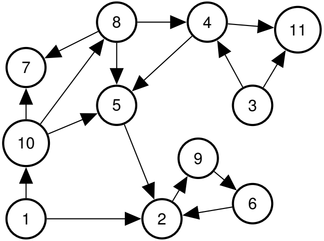

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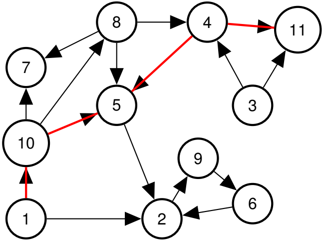

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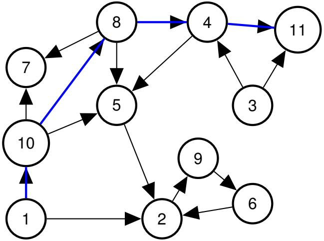

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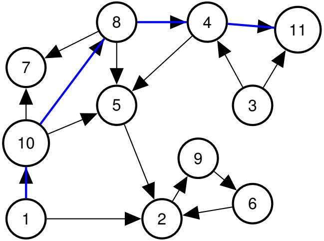

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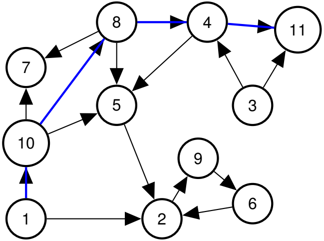

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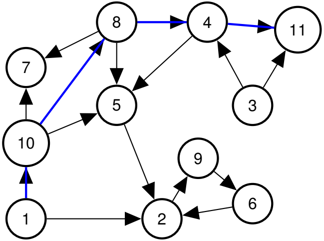

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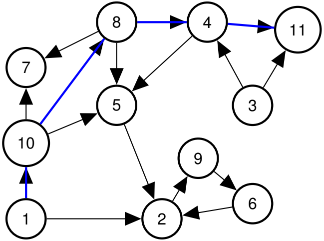

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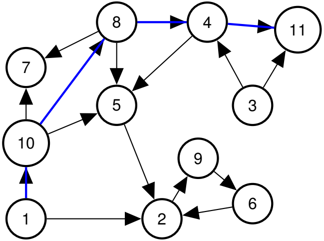

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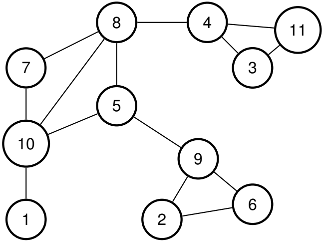
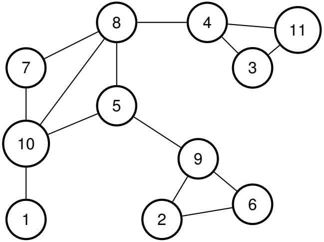
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



u and *v* are connected if there is a path between *u* and *v*.

Connectivity: undirected graph.



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A connected graph is a graph where all pairs of vertices are connected.

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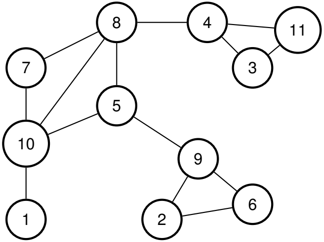

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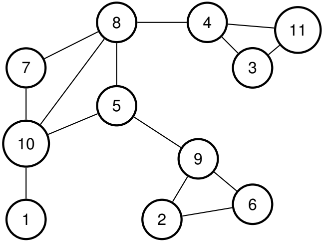

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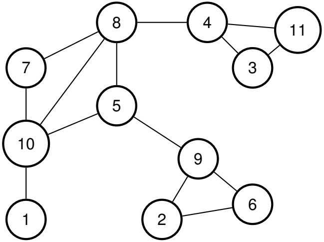

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Is graph connected? Yes?

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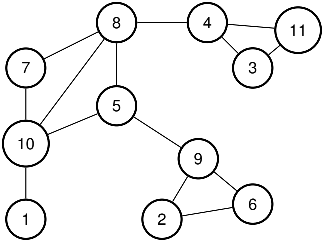

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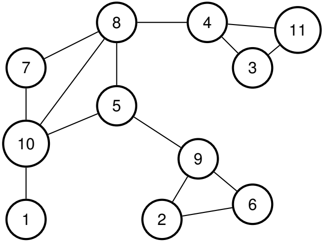

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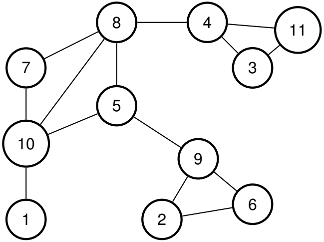
Connectivity: undirected graph.



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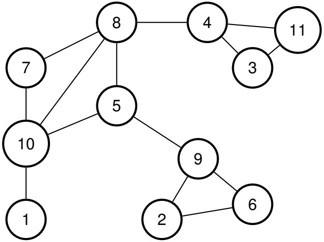
Proof: Use path from *u* to *x* and then from *x* to *v*.

Connectivity: undirected graph.



u and *v* are connected if there is a path between *u* and *v*.

A connected graph is a graph where all pairs of vertices are connected.

If one vertex *x* is connected to every other vertex.
Is graph connected? Yes? No?

Proof: Use path from *u* to *x* and then from *x* to *v*. □

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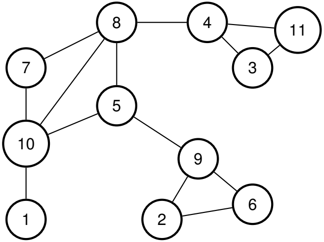

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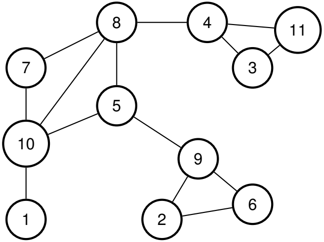

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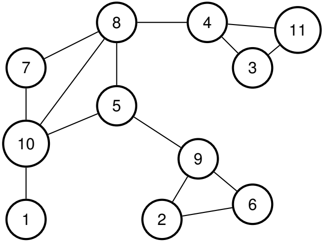

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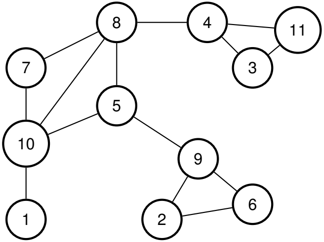

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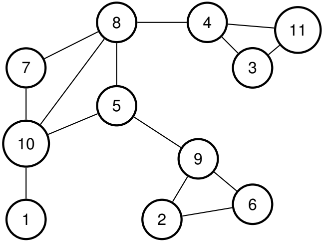

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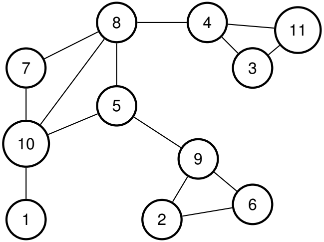
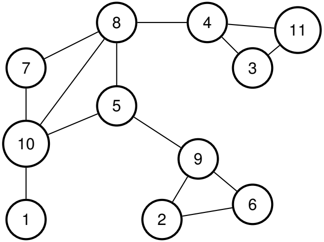
Connected Components: Quiz.



Is graph above connected?

Connected Components: Quiz.



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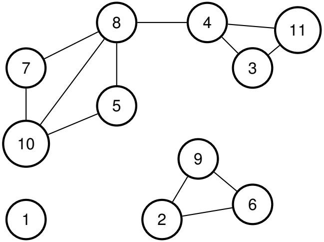

Is graph above connected? Yes!

How about now?

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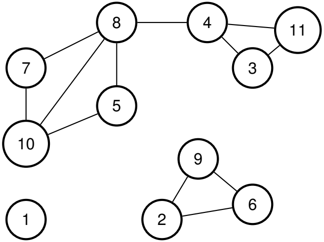

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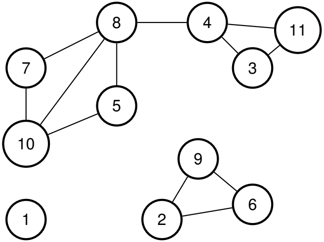

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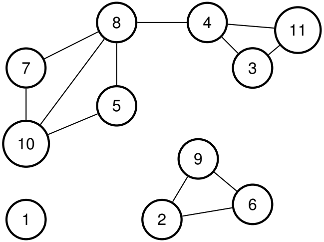

Is graph above connected? Yes!

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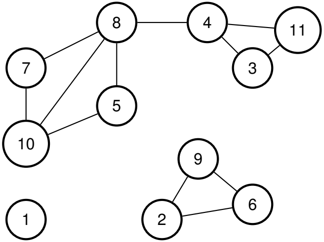
Connected Components? $\{1\},\{10,7,5,8,4,3,11\},\{2,9,6\}$.

Connected Components: Quiz.



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Connected component - maximal set of connected vertices.

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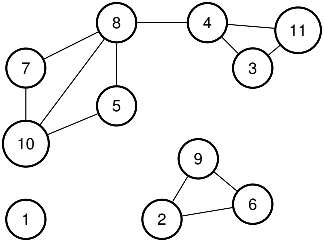

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Quick Check: Is $\{10, 7, 5\}$ a connected component?

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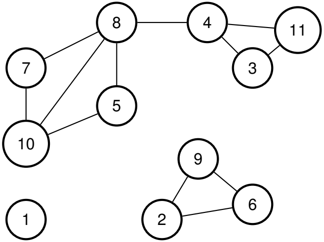

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Connected component - maximal set of connected vertices.
Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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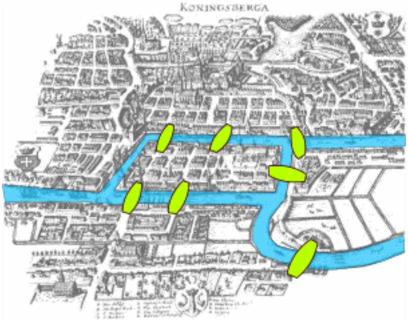

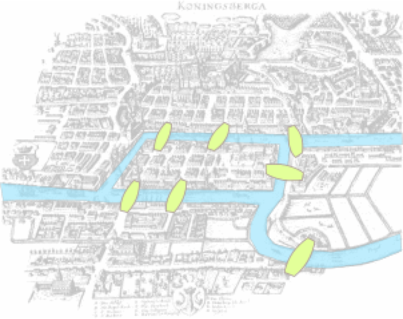

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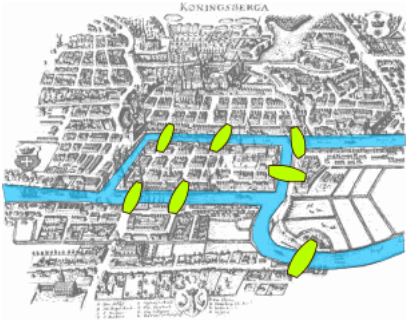

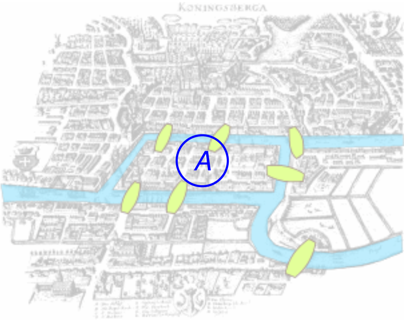

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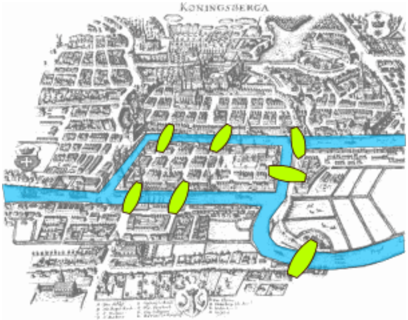

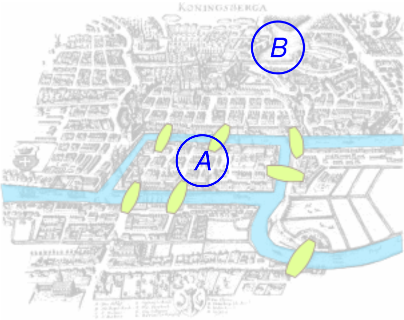

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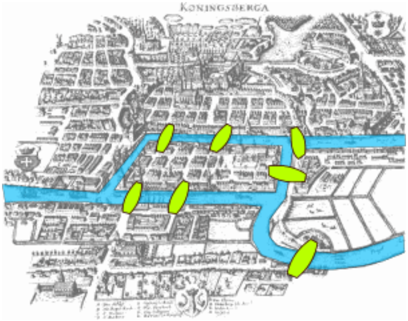

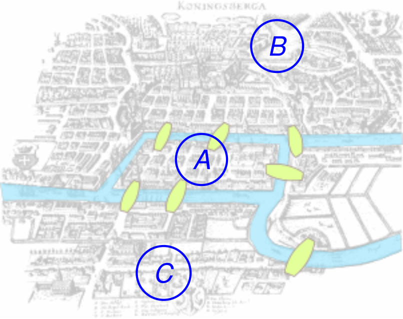

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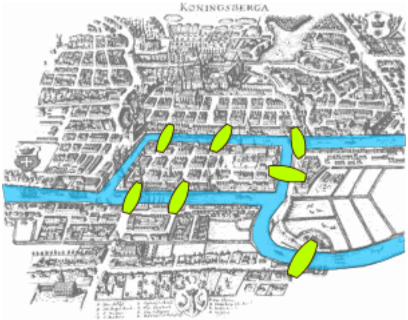

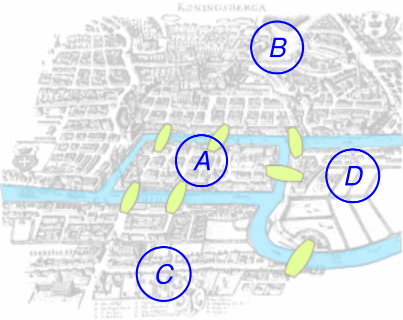

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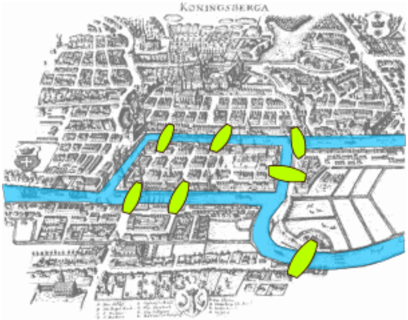

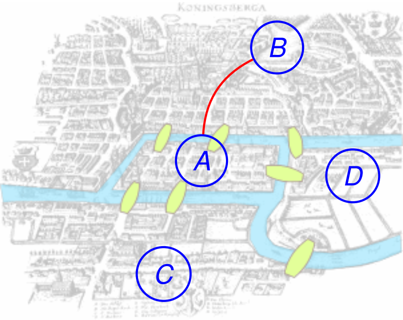

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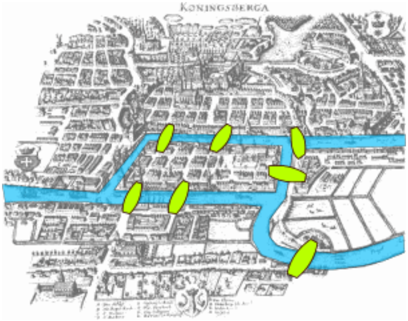

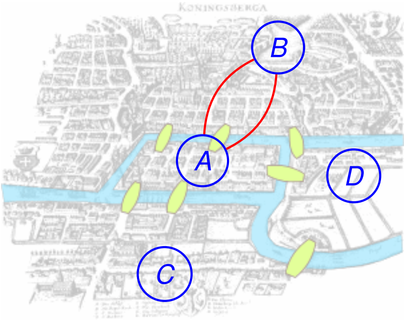

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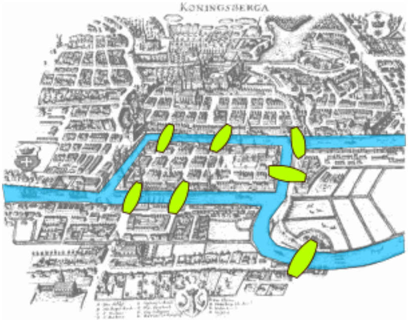

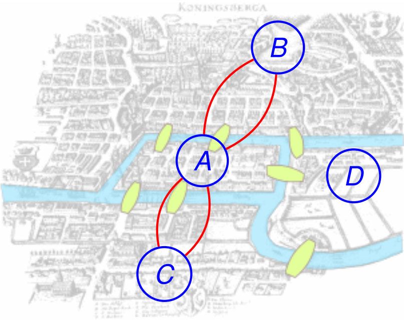

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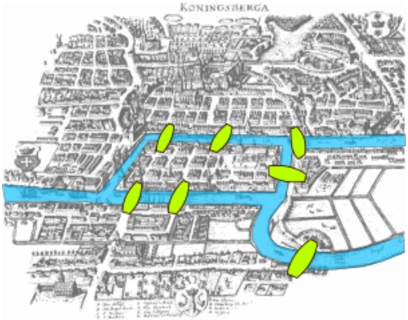

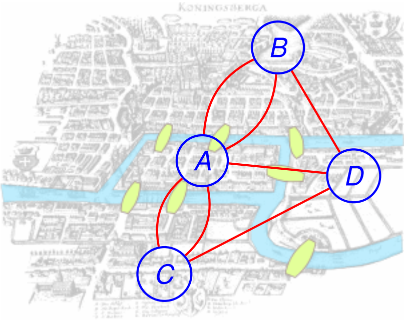

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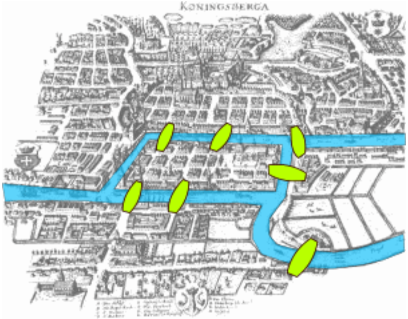

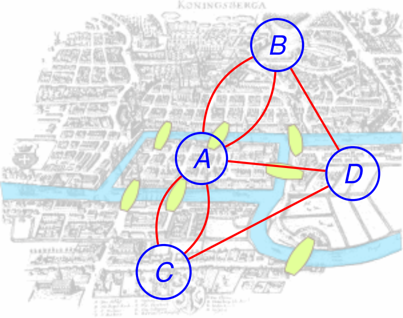

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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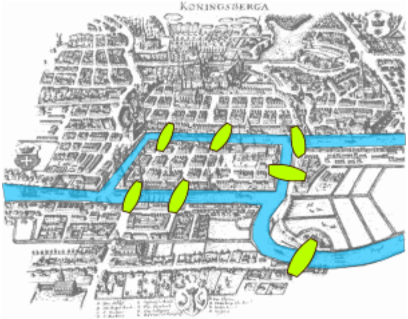

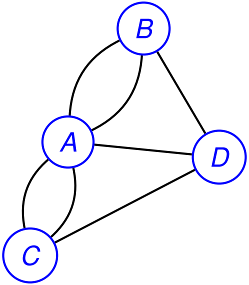

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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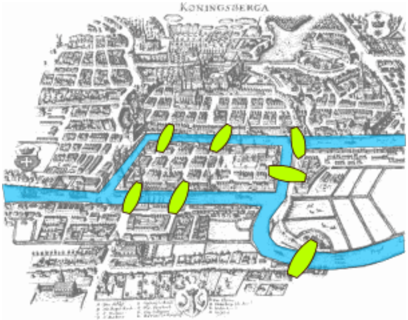

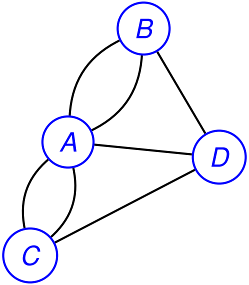

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

Konigsberg bridges problem.

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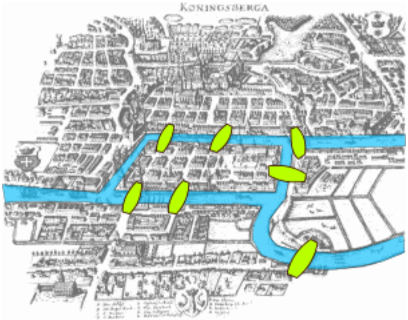

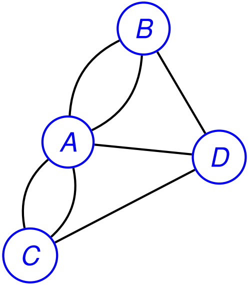

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Eulerian Tour

Eulerian Tour visits every vertex using each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

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Proof of only if: Eulerian $\implies$ connected and all even degree.

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Eulerian Tour is connected so graph is connected.

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Tour enters and leaves vertex v on each visit.

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Tour enters and leaves vertex v on each visit.
Uses two incident edges per visit.

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Therefore v has even degree.

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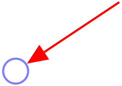

When you enter,

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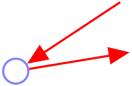

When you enter, you can leave.

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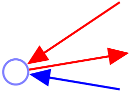

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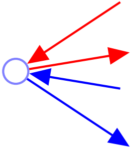

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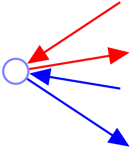

When you enter, you can leave.
For starting node,

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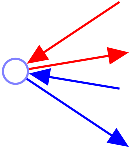
Uses two incident edges per visit. Tour uses all incident edges.
Therefore v has even degree. □



When you enter, you can leave.
For starting node, tour leaves first

Eulerian Tour

Eulerian Tour visits every vertex using each edge exactly once.

Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
Proof of only if: Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex v on each visit.
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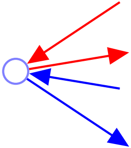

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For starting node, tour leaves first ....then enters at end.

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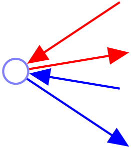

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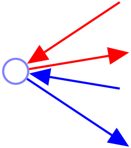

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Not The Hotel California.

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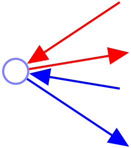

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Not The Hotel California.

(Timestamp: 4:02).

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
We will give an algorithm.

Finding a tour!

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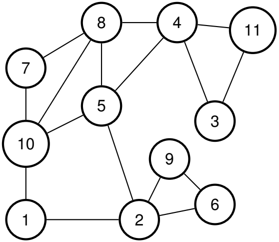

1. Take a walk starting from v (1) on "unused" edges

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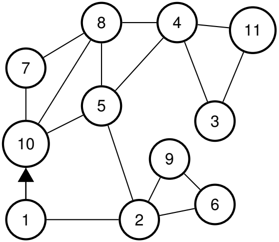

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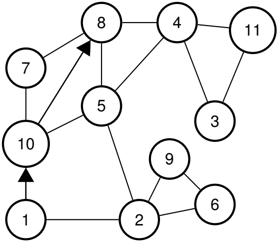

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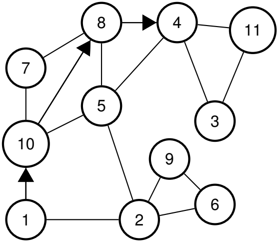

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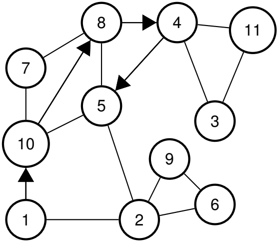

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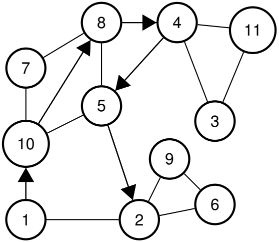

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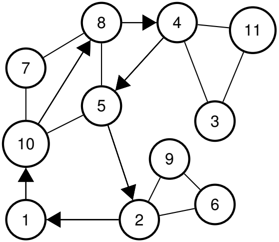

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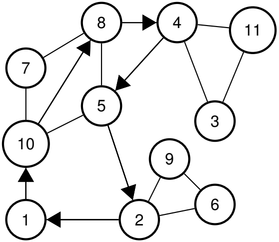

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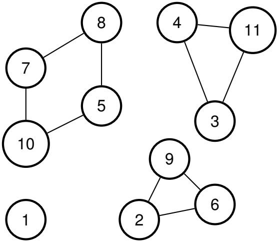

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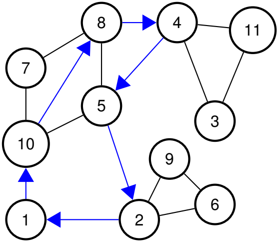

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   Each is touched by C.

Finding a tour!

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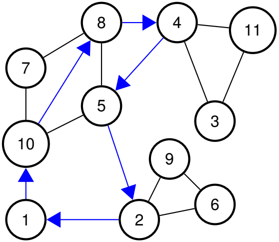

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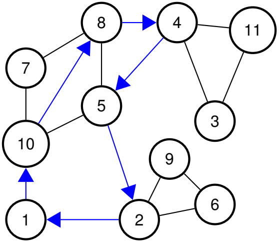

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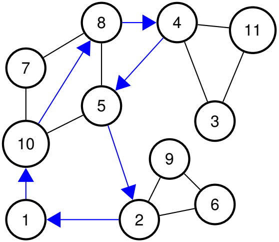

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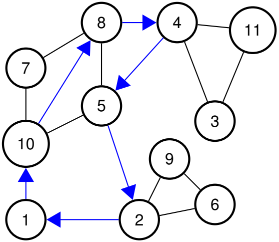

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    Example: $v_1 = 1$,

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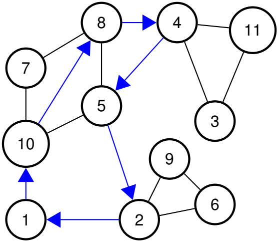

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    Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

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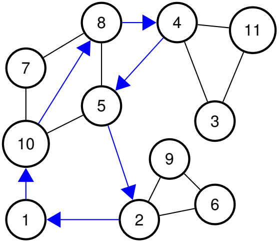

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

Finding a tour!

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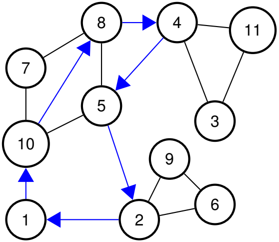

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Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.

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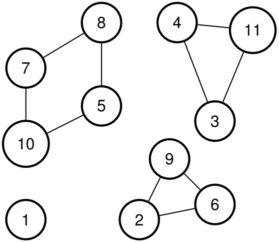

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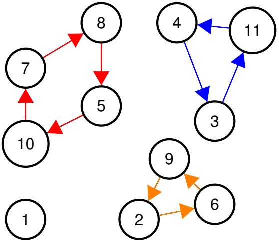

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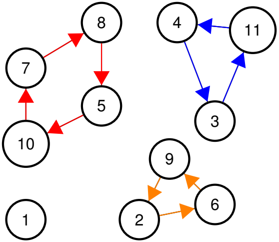

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4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.

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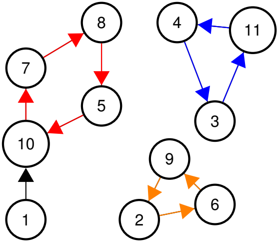

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   1,10

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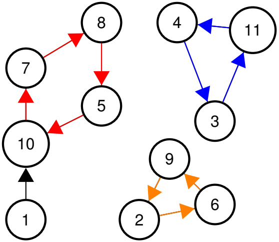

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   1,10,7,8,5,10

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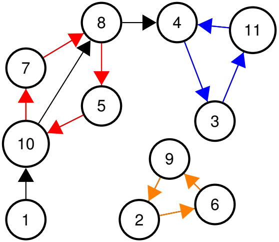

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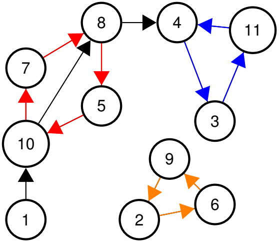

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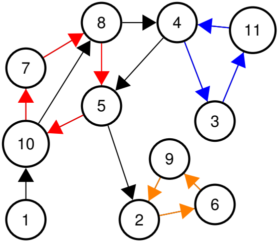

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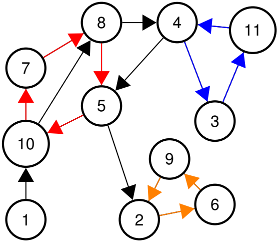

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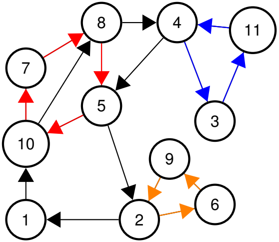

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Proof of Claim: Even degree.

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2. Remove cycle, C, from G.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.

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Claim: Each vertex in each G_i has even degree and is connected.

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Visits edges in C exactly once.
By induction for all edges in each G_i.

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Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, tours uses edges at most once, but may involve a vertex several times.

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(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with a Eulerian tour is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges E' in a connected graph, every connected component is incident to an edge in E'
(E) If one walks on new edges, starting at v, one must eventually get back to v.
(F) Removing a tour leaves a connected graph.

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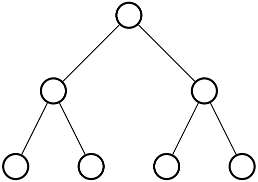
Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.

Binary Tree!



More generally.

Trees.

Definitions:

Trees.

Definitions:

A connected graph without a cycle.

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A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.

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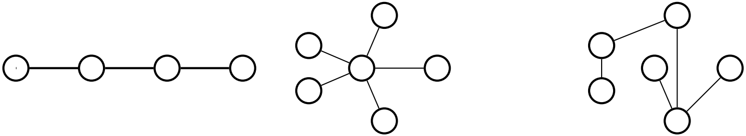

no cycle and connected?

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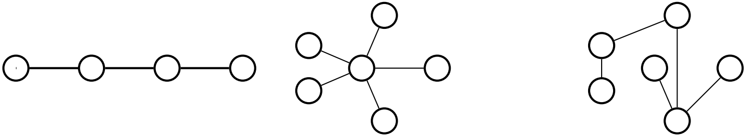

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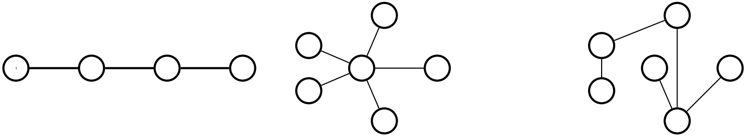

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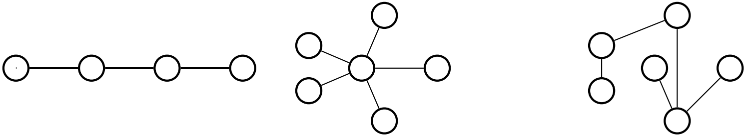

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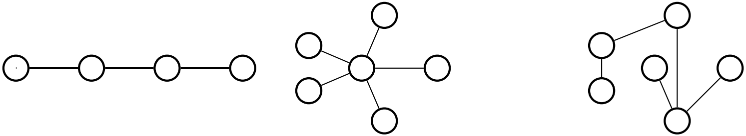

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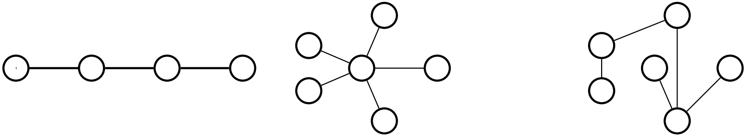

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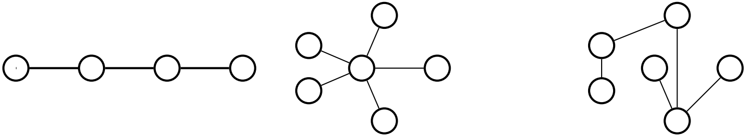

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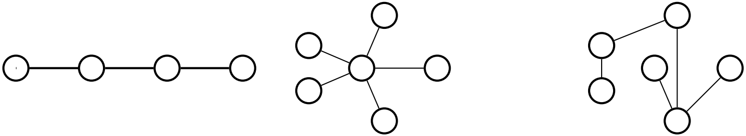

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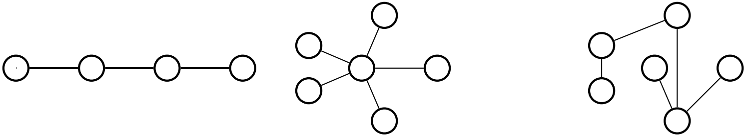

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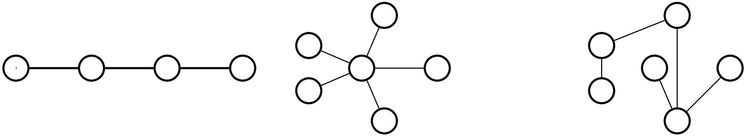

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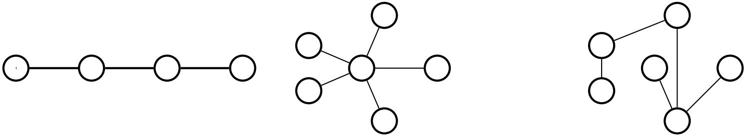

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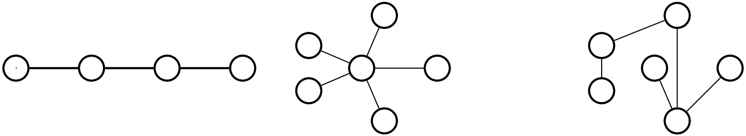

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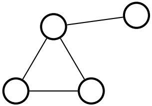
To tree or not to tree!



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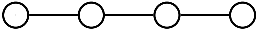

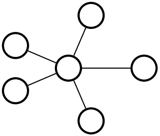

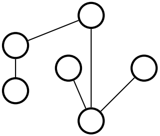

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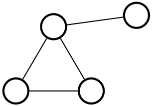

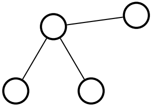

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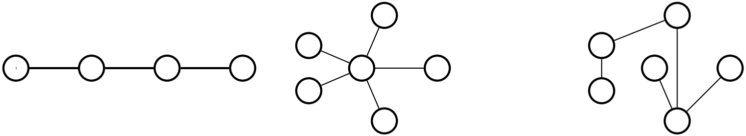

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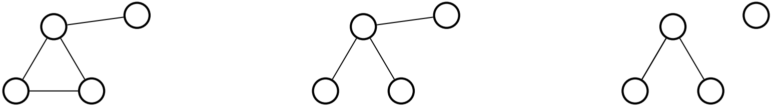

Equivalence of Definitions.

Theorem:
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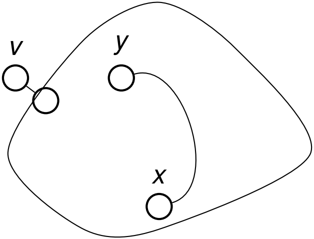

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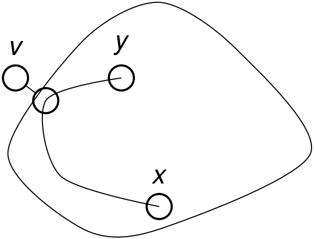
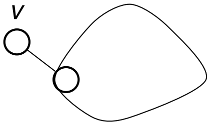


Proof of only if.



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Proof of only if.

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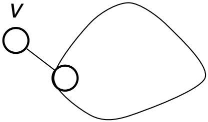
Proof of $\implies$: By induction on $|V|$.



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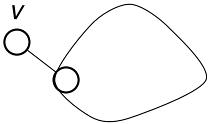

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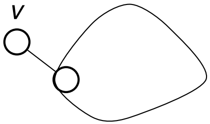

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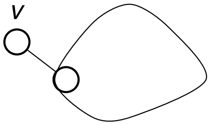

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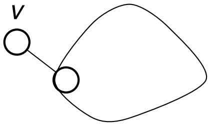

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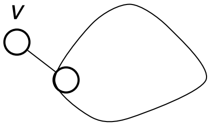

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Proof: First, connected $\implies$ every vertex degree ≥ 1.

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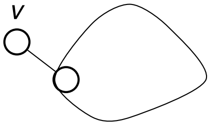

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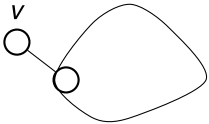
Sum of degrees is $2|E|=2(|V|-1)=2|V|-2$

Proof of only if.



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Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
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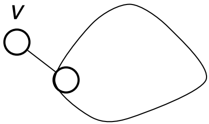

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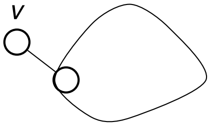

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Cuz not everyone is bigger than average!

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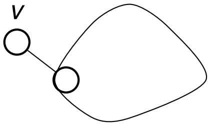

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Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
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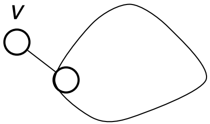

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By degree 1 removal lemma, $G-v$ is connected.

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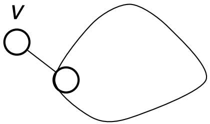

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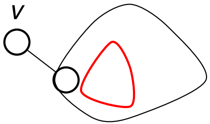

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By degree 1 removal lemma, $G - v$ is connected.
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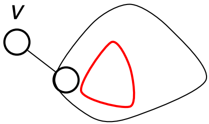
Proof of only if.



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And no cycle in G since degree 1 cannot participate in cycle.

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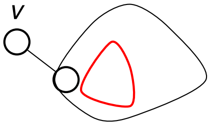

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“G is connected and has no cycles”
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Thm:
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Proof:
Walk from a vertex using untraversed edges.

Proof of if

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Until get stuck.

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Proof of Claim:
Can't visit more than once since no cycle.

Proof of if

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Thm:
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Removing node doesn't create cycle.

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Removing node doesn't create cycle.
New graph is connected.

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Removing degree 1 node doesn't disconnect from Degree 1 lemma.

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By induction $G - v$ has $|V| - 2$ edges.

Proof of if

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G has one more or $|V|-1$ edges.

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Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ **edges.**

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2 - 2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V|-1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
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(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

(B), (C), (D) are true

Lecture Summary.

Graphs.

Lecture Summary.

Graphs.
  Basics.

Lecture Summary.

Graphs.

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Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.

Lecture Summary.

Graphs.
- Basics.
- Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
  - Connected Component.

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Algorithm for Eulerian Tour.

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- Take a walk until stuck to form tour.

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Trees: degree 1 lemma $\implies$ equivalence of several definitions.

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Trees: degree 1 lemma $\implies$ equivalence of several definitions.
 G: n vertices and $n-1$ edges and connected.
  remove degree 1 vertex.
 $n-1$ vertices, $n-2$ edges and connected $\implies$ acyclic.
  (Ind. Hyp.)
 degree 1 vertex is not in a cycle.
 G is acyclic.

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